Delta Science Education

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HKDSE Physics

Core 2: Force and Motion

Chapter 5: Momentum

Part 1

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1. Momentum

(1) Definition of Momentum

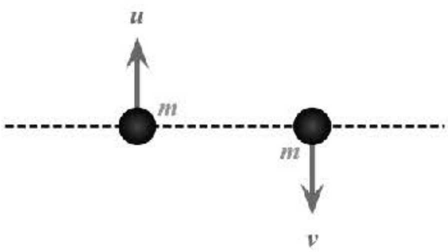
- The linear momentum of an object is

Unit: $\mathrm{kg\ m\ s^{-1}}$ or $\mathrm{N\ s}$

- Momentum is a **vector** which has the same direction as the **velocity**.

Examples that you must fully understand

1. A stone is thrown vertically upwards and it finally falls back to the starting point. Assume the air resistance is negligible. Which of the following statements concerning the stone at the final point compared with that at the starting point is / are true?



   (1) The acceleration of the stone is the same.
   (2) The potential energy of the stone is the same.
   (3) The kinetic energy of the stone is the same.
   (4) The momentum of the stone is the same.

2. State whether each of the following statements is true or false.

   (1) When two bodies of different masses are released from the same height above the ground, they have the same momentum just before hitting the ground.

   (2) When a body is released from rest, it falls down under gravity with its momentum remains unchanged.

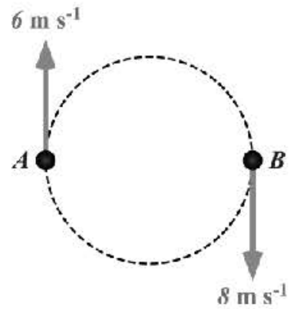
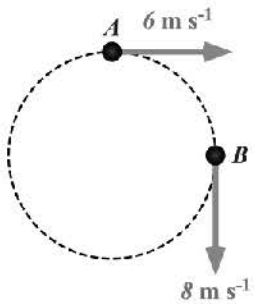
   (3) When a ball is rebounded from the wall with the same speed, its final momentum is same as the momentum before collision.

3. Two objects A and B of mass $4\ \mathrm{kg}$ and $3\ \mathrm{kg}$ respectively have the same momentum. What is the ratio of the kinetic energy of A to that of B?

(2) Change of momentum (subtraction of vectors)

- Similar to change of velocity, change of momentum, also called ______, is found by:

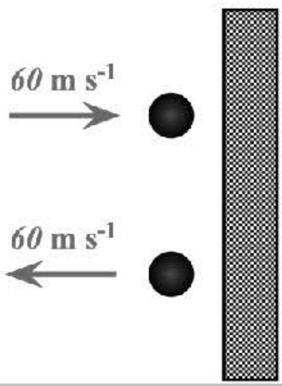
[ ]

e.g. A car of mass **2500 kg** travels from **A** to **B** along a circular path as shown.	
1-D situation	**2-D situation**
	

Examples that you must fully understand

4. Calculate the change of momentum in the following cases:

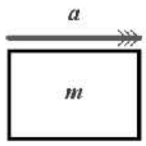
(a) A vehicle of mass 2,000 kg travelling in straight line decreases its speed from 20 m s^{-1} to 5 m s^{-1}.



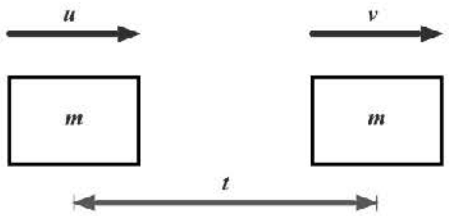
(b) A ball of mass 200 g hitting the wall with a velocity of 60 m s^{-1} rebounds with the same speed. Take the direction towards the left as positive.

(3) Newton's Second Law

- Whenever a body experiences a **net force**, there must be **an acceleration**. **Newton's Second Law** states the relationship between the net force and the acceleration.



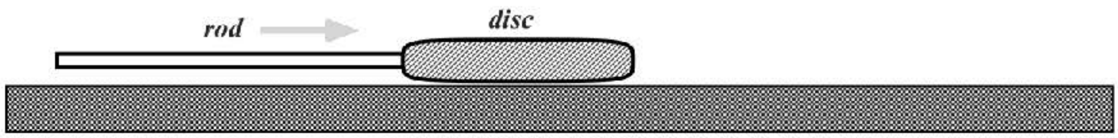
- Whenever a body experiences a **net force**, there must be **a change of momentum in a certain period of time**. **Newton's Second Law** also states the relationship between the net force and the change of momentum.



- Therefore, net force/resultant force/unbalanced force can also be called "**rate of change of momentum**".

Examples that you must fully understand

5. In a table game, a small disc of mass $0.12\ \text{kg}$ is set to move on a frictionless horizontal table by a strike of a rod as shown in the figure below. The average force exerted by the rod on the disc is $60\ \text{N}$ and the duration time of contact between the rod and the disc is $5\ \text{ms}$.

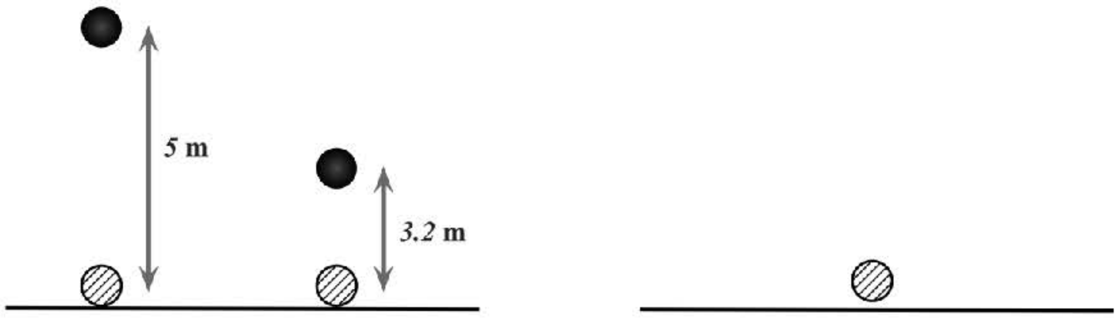


(a) What is the speed of the disc on leaving the rod?

(b) What is the work done on the disc by the player?

Examples that you must fully understand

6. A ball of mass 0.2 kg is dropped from a height of 5 m and rebounds to a height of 3.2 m.

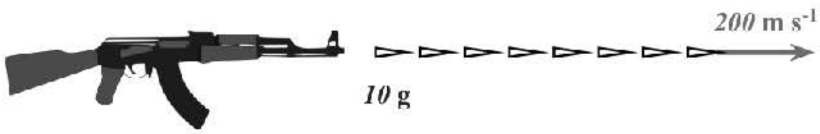


(a) Find the speed of the ball v_1 just before hitting the ground v_2 just after hitting the ground.

(b) If the time of contact with the ground is 0.5 s, find the average net force acting on the ball during the impact.

(c) Hence find the impact force acting on the ball by the ground during the collision.

7. A rifle of mass 0.5 kg fires bullets each of mass 10 g with a muzzle velocity of 200 m s^{-1}. If the bullets are fired at a rate of 100 shots per second, find the average force acting on the rifle. State the direction as well.



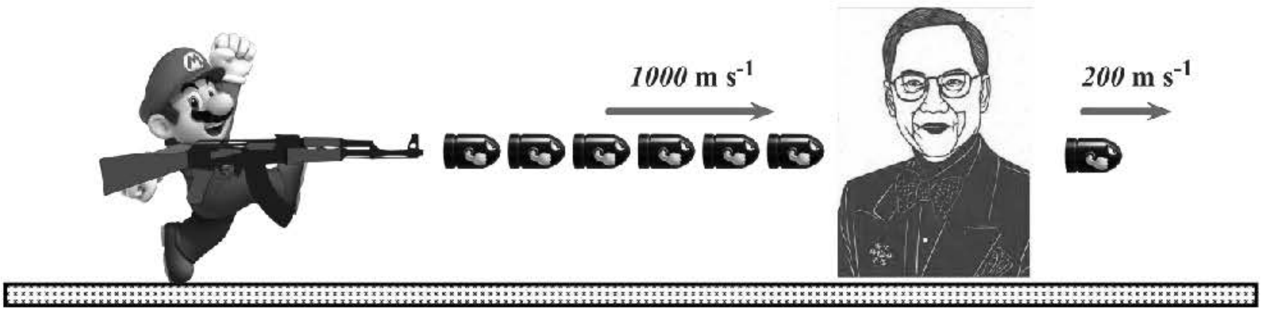
Examples that you must fully understand

8. A rocket of initial mass 3.84×10^5 kg ejects fuel at a rate of 1500 kg in 1 second with a speed of 3200 m s^{-1}.

(a) Find the thrust acting on the rocket in the first second.

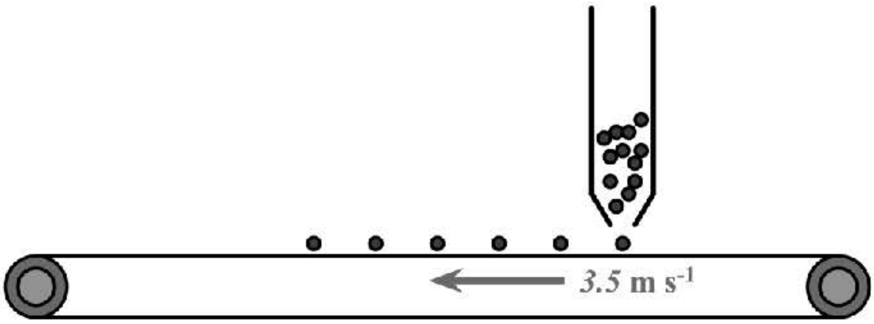
(b) Determine the initial acceleration of the rocket.

9. A soldier standing firmly on the ground fires a machine gun held in his hands. Six bullets, each of mass 0.15 kg, are fired from the gun every second. The bullets travel horizontally to the right and hit Donald Tsang at a speed of 1000 ms^{-1} and leave at 200 ms^{-1}. What is the magnitude and the direction of the force acting on Donald?



Examples that you must fully understand

10. A conveying belt is moving with a uniform velocity of 3.5 m s^{-1}. Sand from a funnel drops onto the moving belt continuously at a rate of 2.4 kg min^{-1}. When the sand drops onto the belt, it takes some time to accelerate until it attains the same velocity as the moving belt. During this time, the belt has moved a distance of 14 m. Assume the sand undergoes constant acceleration before it reaches the speed of the belt.



(a) After the sand drops onto the belt,

(i) Which force acts on the sand to give its acceleration? ☐

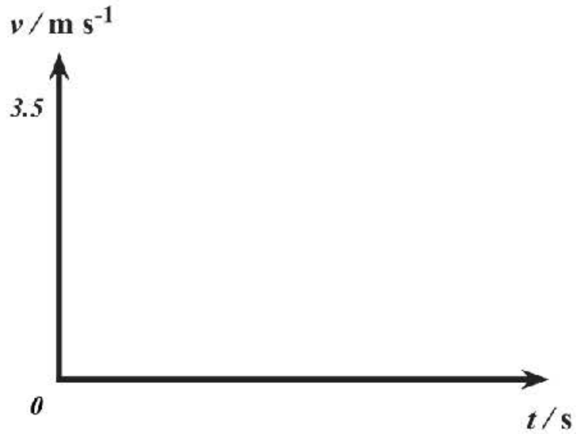
(ii) What is the direction of this force? ☐

(iii) Find the magnitude of this force.

(b) Find the horizontal force required to keep the belt move at the uniform velocity of 3.5 m s^{-1}.

Horizontal force to keep the belt move = ☐

(c) What is the distance moved by the sand during the period of acceleration?



(d) Determine the acceleration of the sand before it reaches the velocity of the belt.

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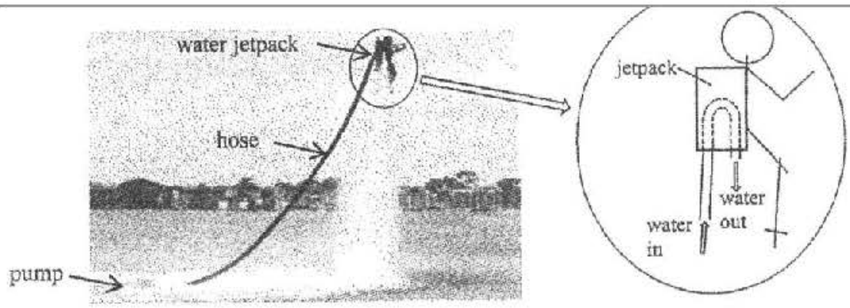
11. A person wears a water jetpack which enables him to stay 'afloat' in equilibrium in the air as shown in the above figure. A pump on the sea surface continuously pumps water to the jetpack via a hose and the water is then ejected downwards.



(a) Referring to the above figure, water enters the U-shape hose inside the jetpack with a certain speed and is then ejected out vertically downwards. Use Newton's law(s) of motion to explain why a lifting force acting on the person is produced.

By Newton's second law,

By Newton's third law,

(b) Suppose that water enters the jetpack with a speed of 10 m s^{-1} vertically upwards and is then ejected out at the same speed vertically downwards. Take g = 10 m s^{-2}.

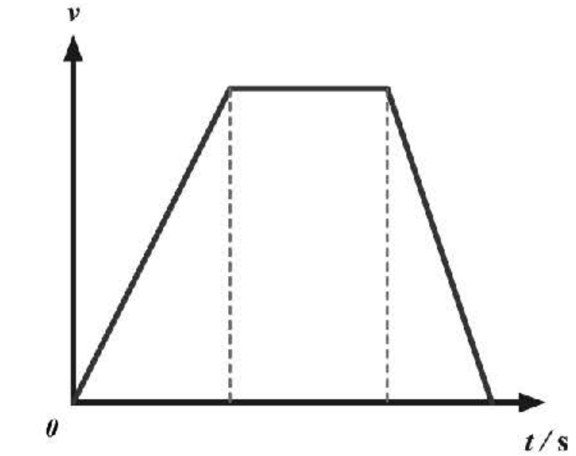
(i) Just by considering the change of momentum of water, estimate how much water, in kg, has to be ejected per second to support a 100 kg person. (Neglect the weight of the hose.)

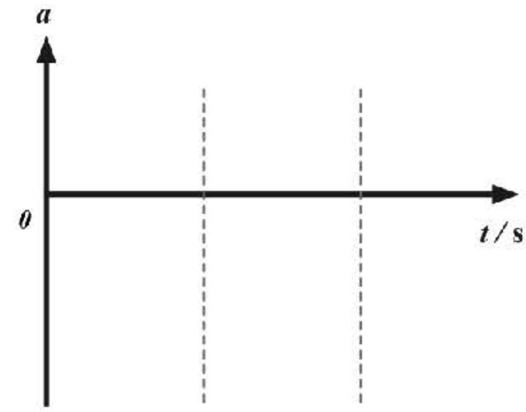
(ii) Water is pumped continuously to the water jetpack at a height of 7.5 m above sea surface and then ejected from it. By considering the gain in mechanical energy of the water, estimate the minimum output power of the pump.

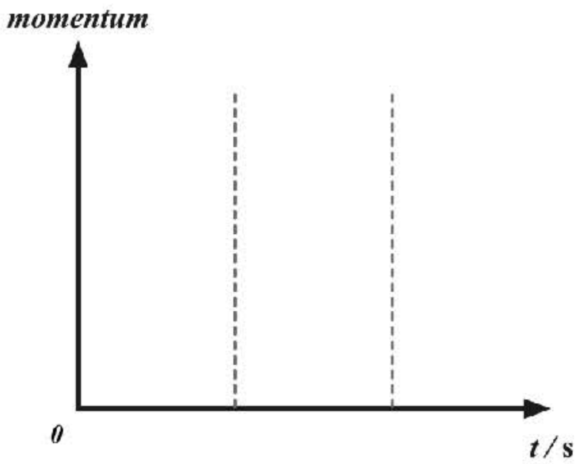
(c) Suppose the person wants to go up to a higher position. If the speed by which water enters and is ejected from the jetpack remains the same, would the amount of water ejected per second be greater than, equal to or smaller than the result found in (b) (i) for him to stay "afloat" in equilibrium at a higher position? Explain.

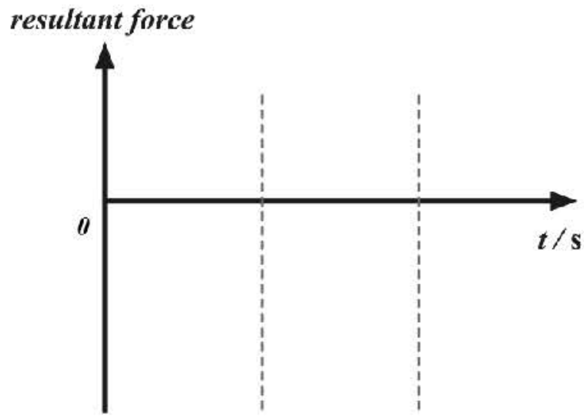
To stay afloat in equilibrium, ______ lifting force is required. Thus, the amount of water ejected is the ______.

(4) Time variation graphs of force and momentum



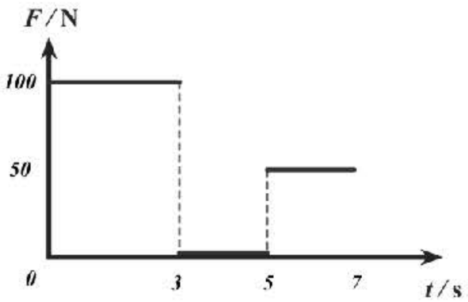


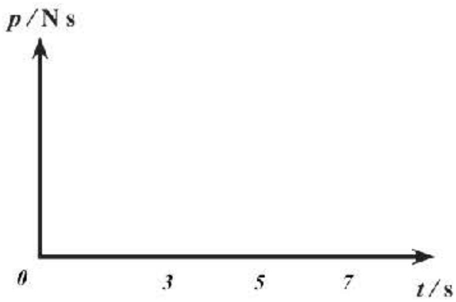




Examples that you must fully understand

12. A ball of mass $10\ \text{kg}$ moving at an initial speed of $5\ \text{m s}^{-1}$ is subjected to a force as shown in the F-t graph.



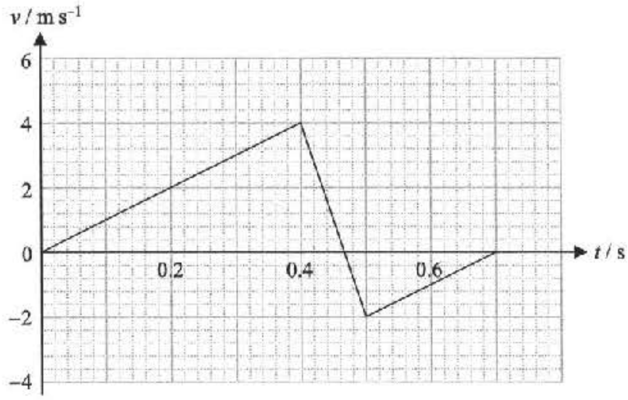


Sketch the variation of the momentum of the ball.

Examples that you must fully understand

13. A rocket ejects fuel so that its mass decreases at a constant rate. If the acceleration of the rocket keeps constant and it starts from rest, sketch the variation of the resultant force F acting on the rocket with time t; and the variation of the momentum p of the rocket with time t.

14. A ball of mass 0.2 kg is released from rest. It hits the ground and rebounds. The velocity-time graph of the ball is shown above. Which of the following statements are correct?

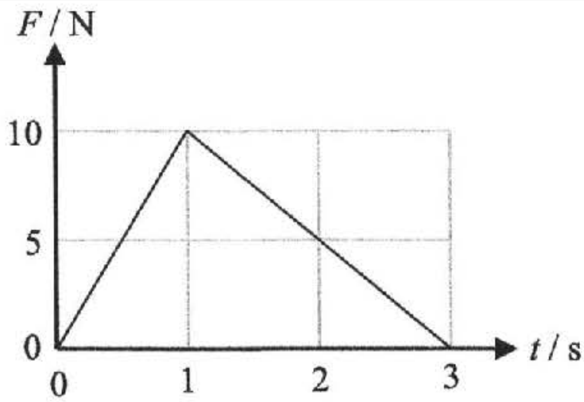


(1) The magnitude of the change in momentum of the ball during the collision is 1.2 kg m s^{-1}.

(2) The magnitude of the average force acting on the ball by the ground during the collision is 12 N.

(3) There is mechanical energy loss during the collision.

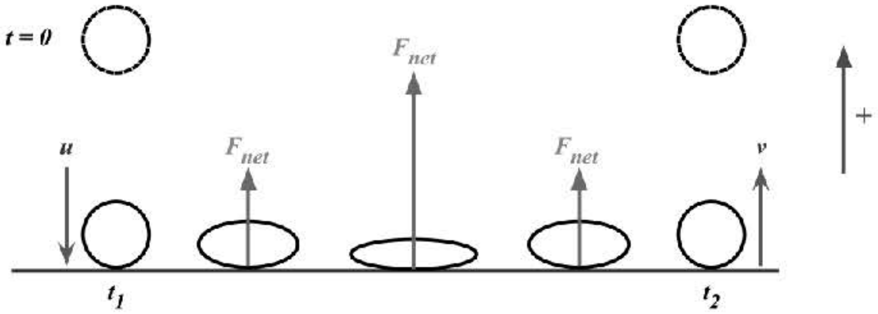
15. An object of mass 3 kg is initially at rest on a smooth horizontal ground. A force F is applied horizontally to the object such that the magnitude F varies with time t as shown. What is the speed of the object at t = 3 s? Neglect air resistance.



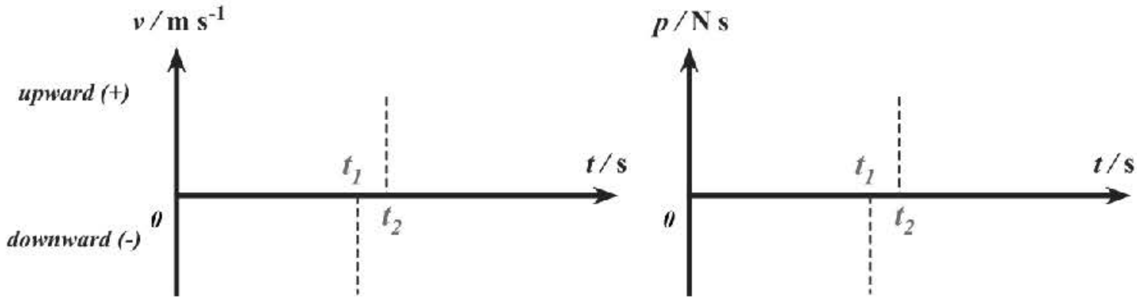
2. Impact

(1) Impact

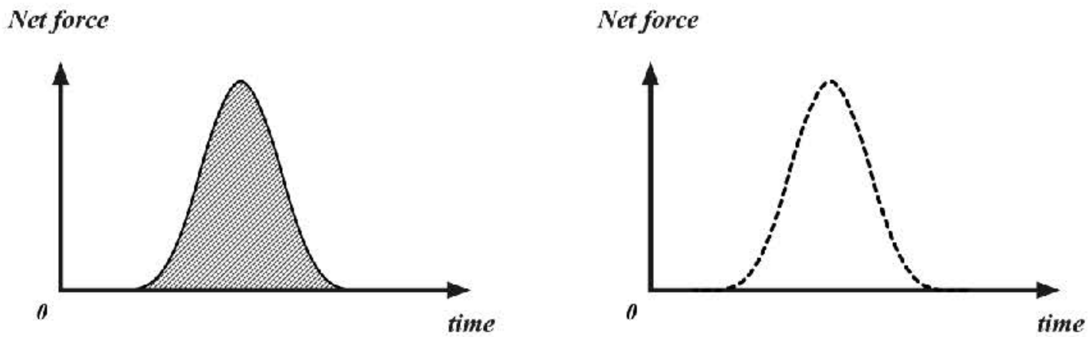
- Deformation of an elastic ball during impact:



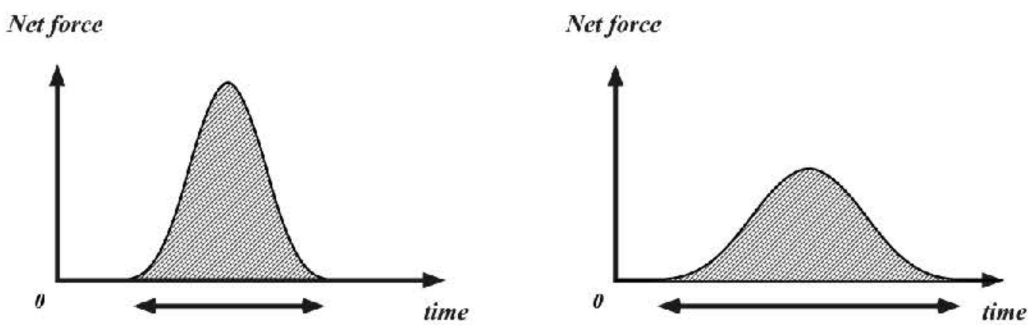
- Variation of the velocity and momentum of the ball with time:



- Time variation of the net (impact) force on the ball by the ground:



- For the **same change of momentum**, **longer time of impact** gives the **smaller average net (impact) force**.



Examples that you must fully understand

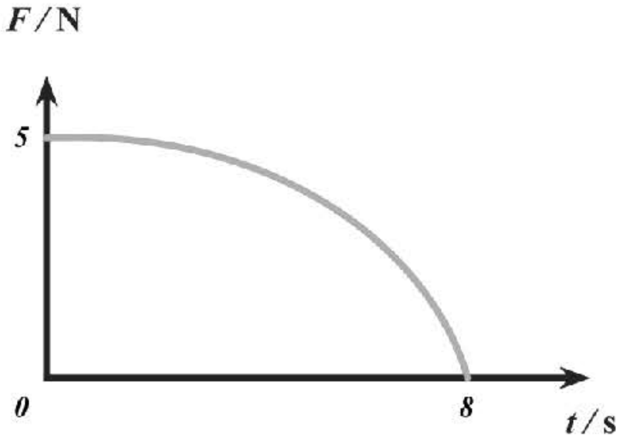
16. A ball of mass 0.25 kg hits a vertical wall with a horizontal speed of 30 m s^{-1}. It rebounds with a speed of 20 m s^{-1}. Find the impulse exerted by the wall on the ball. (Take the initial direction as positive.)

17. A football with initial momentum of 3 kg m s^{-1} is stopped by a goalkeeper in 0.01 s. Calculate the average force applies on the ball. (Take the initial direction as negative.)

18. A car of mass 1000 kg with a velocity of 15 m s^{-1} and accelerates to 20 m s^{-2}. Calculate the impulse on the car. (Take the initial direction as positive.)

19. In a baseball game, a pitcher throws a 1 kg baseball with speed 5 m s^{-1} and the ball is hit by a batsman with impulse of 15 N s. Find the final speed of the ball. (Take the initial direction as negative.)

20. The graph shows the time variation of the net force, F, acting on an object of mass 2 kg. The object is initially at rest and its subsequent motion is along a straight line. Which of the following statements is / are correct?



(1) The object is undergoing uniform acceleration in this period.

(2) At the instant when the force F drops to zero, the object becomes at rest.

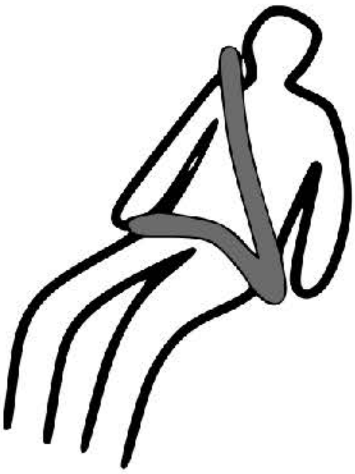
(3) The momentum of the object is conserved throughout the motion.

(4) The object attains maximum acceleration at $t = 0$ s.

(5) The maximum velocity of the object is greater than 10 m s^{-1}.

(2) Examples

- Landing: When we jump down from a high place, we must bend our knees to increase the duration time of landing and thus reducing the average impact force during landing.
- Choice of shoes: Air-cushioned shoes can increase the time of impact and reduce the average force whenever the shoes are in contact with the ground.



- Seat belt in vehicles: Seat belt should be elastic to increase the time of impact and thus reduce the average impact force during collision. Seat belt prevents the passenger from being thrown out of the car.
- Air-bags: During accident, the air-bags would explode out. Passengers would then collide with the elastic air-bags to increase the time of impact and thus reduce the average impact force.
- Car bumpers: The car bumpers are elastic to increase the time of impact and thus reduce the average impact force.
- Design of the car body: The front section and rear sections of a car are designed to be crumple (i.e. collapsible) during impact. This can increase the time of impact during collision and thus decrease the average impact force acting on the passengers.

Examples that you must fully understand

21. A car moving at high speed hits a wall as shown in the figure below.

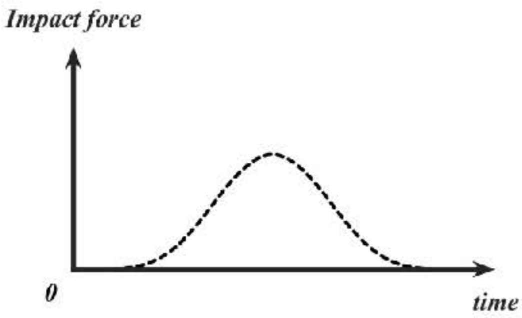
(a) **The driver wears a seat-belt. Which of the following statements is / are correct?**

(1) *The seat-belt can reduce the change in momentum of the passenger during a collision.*

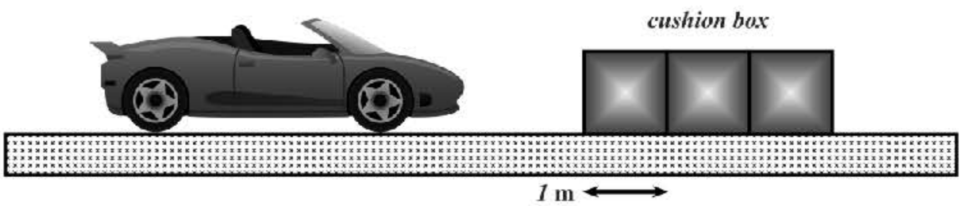
(2) *The seat-belt can reduce the impact force acting on the passenger by increasing the duration time of impact.*

(3) *The seat-belt can prevent the passenger from throwing forward and hitting the front window of the vehicle.*

(b) **The figure on the right shows the time variation of the force acting on the driver. Draw the variation of force if the car is not equipped with an airbag.**



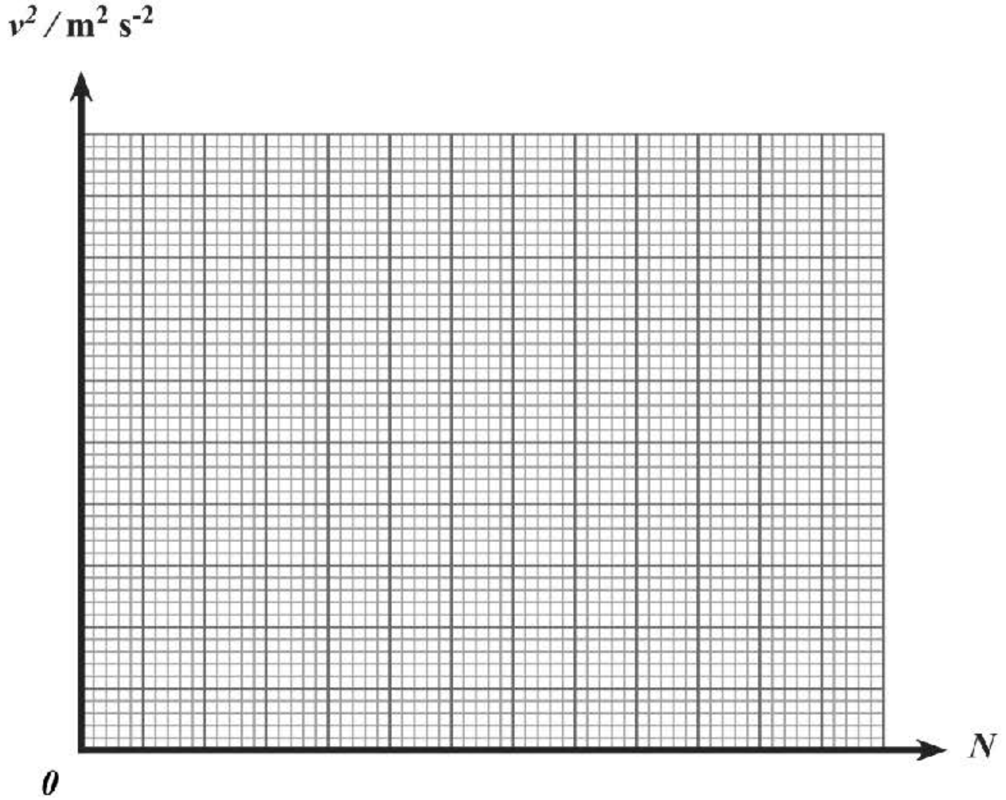
22. The figure below shows a crash cushion system installed at some junctions on highways. The system consists of a number of identical cushion boxes, containing sand or water, lined up and fixed on the road surface. During a crash, the boxes will burst one after another when the car runs through them. The boxes will act as a series of cushions and offer protection to the passengers.



In a pilot test on the cushion boxes, a car of mass **1800 kg** travelling at a speed of 28 m s^{-1} runs through the boxes on a road. Each box has a length of **1 m.** the speed v of the car after running through all the boxes is recorded. The test is repeated by varying the number of boxes N installed in the system. The Table below shows the results obtained. Assume that the deceleration of the car remains unchanged in the test.

Number of boxes N	0	1	2	3	4
v / m s^{-1}	28.0	26.3	24.4	22.4	20.2
v^2 / m^2 s^{-2}	784	692	593	502	408

(a) Using the data in the Table, plot a graph of v^2 against N.



(b) From the graph, determine the minimum number of cushion boxes to stop the car in the test.

Minimum number of boxes to stop the car = ☐

(c) Hence find the average resistive force exerted by the cushion boxes on the car during the collision.

(d) If the test is repeated with a lighter car travelling at the same initial speed, sketch the graph of v^2 against N that you would expect to obtain. Label it as X.

(e) If the test is repeated with the same car but travelling at a greater initial speed, sketch the graph of v^2 against N that you would expect to obtain. Label it as Y.

(f) If the cushion boxes are replaced with concrete blocks, it is found that the above test car would be stopped within 0.2 s. Calculate the average impact force acting on the car. Comment on this force with that of using cushion boxes.

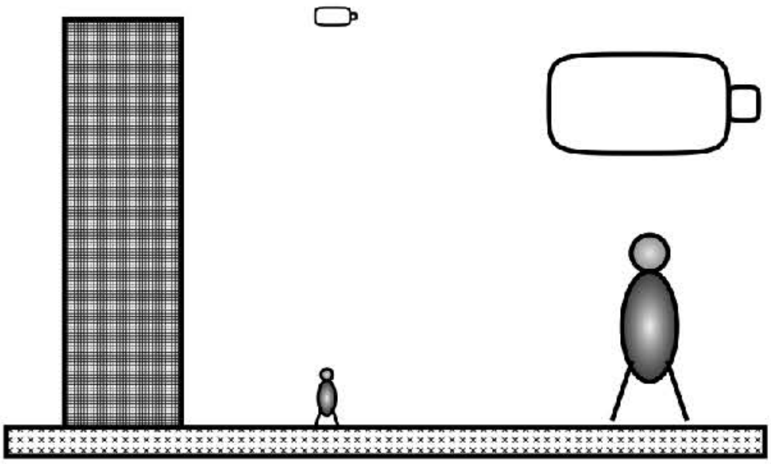
This impact force is ☐ than that of using cushion boxes since the duration time of impact is ☐ for the ☐ momentum change.

Examples that you must fully understand

23. A motor-bike driver has to wear a helmet according to the Hong Kong laws. The interior of helmet is made of spongy material. Explain the importance of the helmet.

The spongy material can increase the time of impact, thus reduce the impact force on the head during an accident.

24. In a government advertisement, a dry cell of mass 65 g is dropped from the 15th floor of a building at a height of 60 m. The dry cell finally hits the head of a girl.



(a) Find the average force acting on the girl by the dry cell during the impact if the duration time of impact is 5 ms.

Average force acting on the girl by the dry cell during the impact = ______

(b) What are the main assumptions in the above calculation?

Assume the ______ of the girl is ______.

Assume the ______ is negligible.

Assume the dry cell ______ ______ from the girl's head.

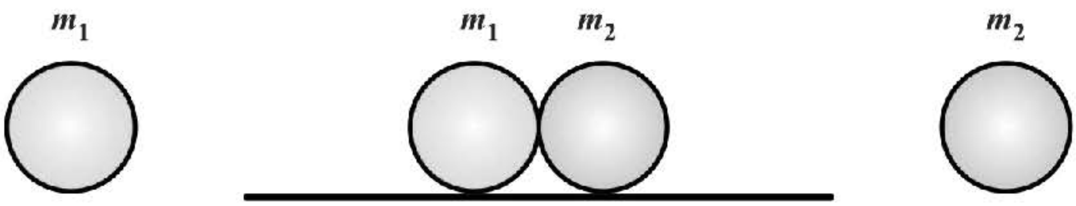
3. Conservation of Momentum

(1) The Law of conservation of momentum

The total momentum of a system remains conserved provided there is no EXTERNAL NET FORCE acting on the system.

(2) Conservation of momentum in a collision

Before collision **After collision**



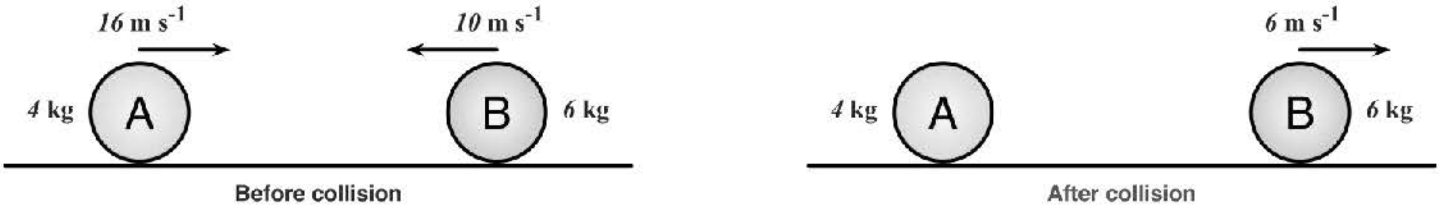
During collision

- According to the Newton's third law, during the impact, each ball experiences the same magnitude of impact force, thus, there is **no external net force** acting on the ______________. The total momentum of the two balls is conserved, i.e. total initial momentum is equal to the total final momentum.

<u>**By Newton's Second Law:**</u> <u>**By Newton's Third Law:**</u>

Examples that you must fully understand

25. Two balls *A* and *B* move towards each other on a smooth horizontal surface as shown. After collision, ball *B* moves away with a velocity of 3 m s^{-1}. Duration time of impact is 0.5 s. take the direction towards the right as positive.



(a) Find the velocity of *A* after impact. State its direction.

(b) Is momentum of ball *A* conserved during the impact? Explain briefly.

[ ]! There is [ ] acting on ball *A* by ball *B* during the impact.

(c) Is the total momentum of the two balls conserved during the impact? Explain briefly.

[ ]! There is [ ] acting on the two balls during the impact.

(d) Which of the following statements is/are correct?

(1) The change of momentum of A is equal in magnitude to the change of momentum of B.

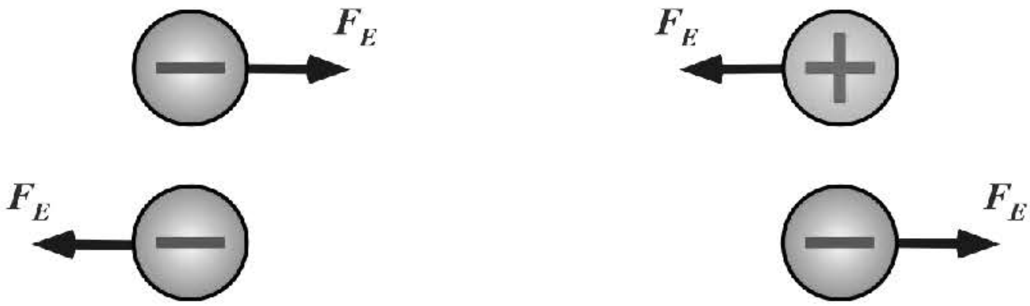
(2) The change of velocity of A is equal in magnitude to the change of velocity of B.

(e) Find the force acting on *A* by *B*; and the force acting on *B* by *A* during the impact. Explain briefly why their magnitude are equal.

They are equal in magnitude since they are [ ].

(3) Transfer of momentum

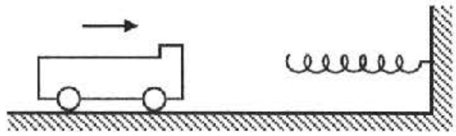
- Momentum cannot be destroyed, dissipated or created. Momentum can only be **transferred** between two bodies.



Examples that you must fully understand

26. A trolley travels with constant velocity to the right on a smooth horizontal ground and collides with a light spring attached to a wall fixed to the ground (Earth) as shown in the figure. At the instant that the trolley comes momentarily to rest during collision, what has happened to the initial momentum of the trolley?

A. ***The initial momentum has been stored in the spring.***
B. ***The initial momentum has been changed into sound and heat.***
C. ***The initial momentum has been destroyed by the friction due to the ground.***
D. ***The initial momentum has been transferred to the Earth.***



27. Considering the above question, at the instant that the trolley comes momentarily to rest during the collision, we cannot observe the Earth to be moving. Thus, is momentum not conserved?

For the trolley, since there is an ________ acting on the trolley, its momentum is not conserved.

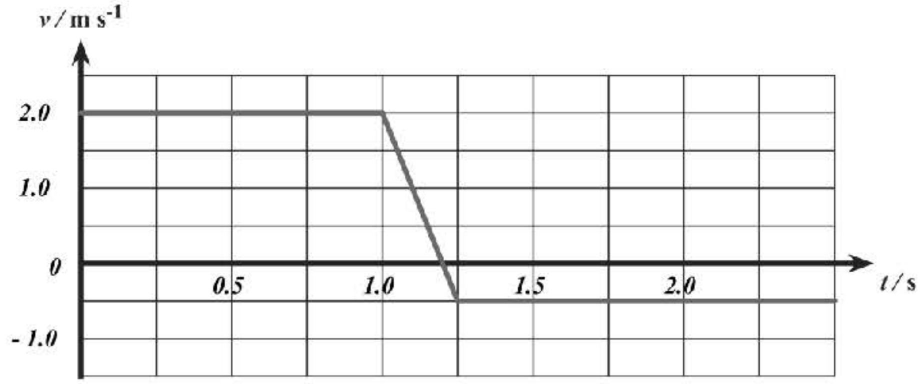
For the trolley and Earth system, since there is ________ acting on the system, the momentum of the trolley and Earth system is conserved. At the instant that the trolley comes momentarily to rest, its momentum is transferred to the ____. However, since the ____ of the Earth is too large, its ____ is negligible.

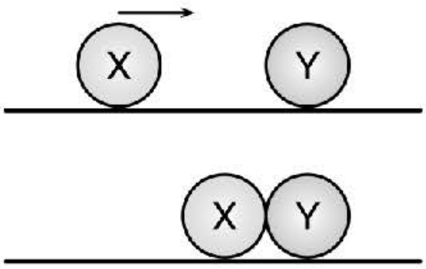
28. In a racing competition, the momentum of each competitor during the race is greater than that before they start running. Which of the below statements is / are correct?

(1) This situation violates the law of conservation of momentum.
(2) The law of conservation of momentum applies only to collisions between two objects.
(3) A force acts on each competitor to increase his momentum as he starts running.

Examples that you must fully understand

29. The figure below shows the variation of the velocity of a ball X of mass $0.2\ \mathrm{kg}$ when it collides with another ball Y which is initially at rest.

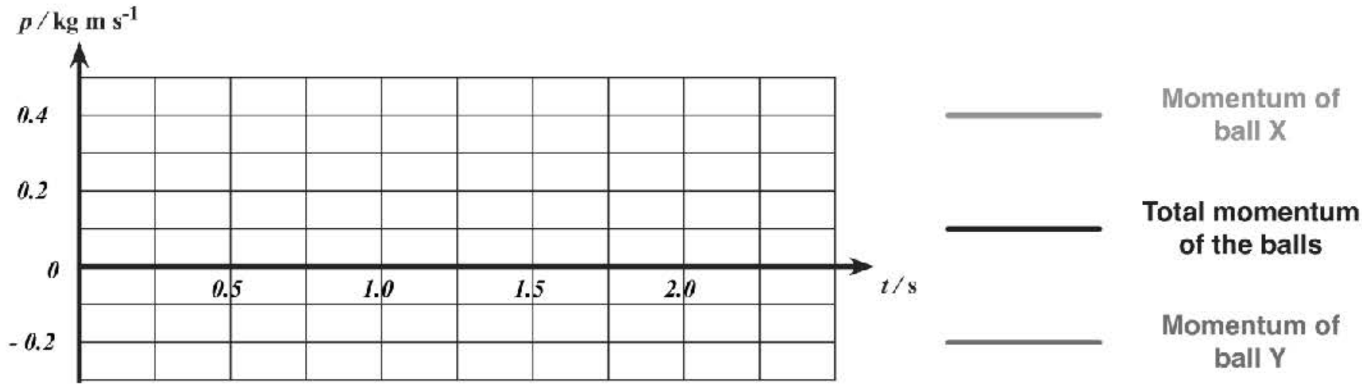




(a) Find the change of momentum of the ball X.

(b) Find the average impact force acting on the ball X.

(c) Sketch the variation of the momentum of ball Y and the total momentum of the two balls on the graphs below.

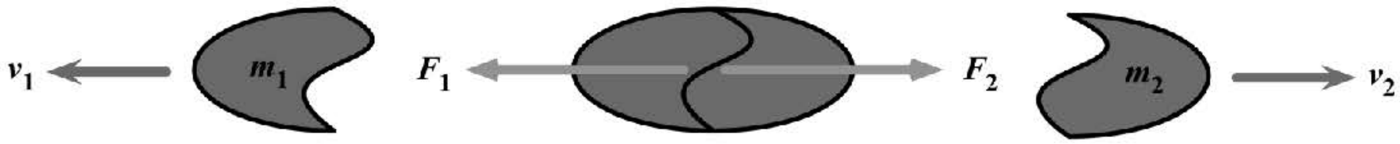


30. Two astronauts, A and B float in the cabin of a space shuttle orbiting round the Earth. Astronaut A is at rest and he carries a tool box while Astronaut B is moving towards A. In order to avoid head-on collision with astronaut B, A throws the toolbox to B and B grasps the toolbox once it reaches him. How can a collision be avoided by doing so?

Since some momentum of A is ______ to B, the ______ of A and B can then be ______ to avoid the occurrence of a collision.

(4) Conservation of momentum in an explosion

- Suppose a stationary body splits into two fragments:



- Since no external net force acts on the system, the total momentum of the system is conserved.

- After the explosion, the total kinetic energy of the system is increased. If the two fragments have different masses, they must have different kinetic energy:

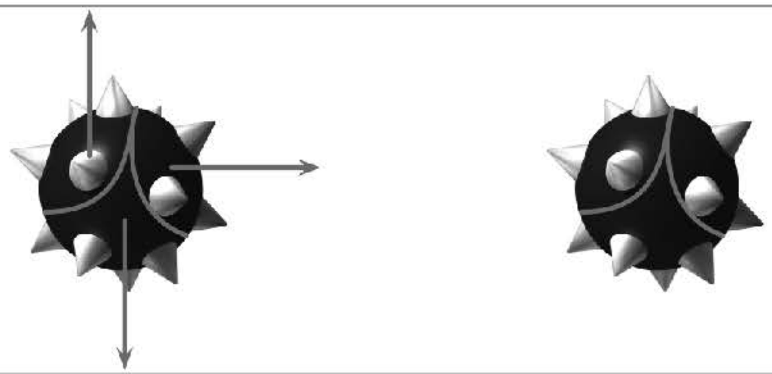
Examples that you must fully understand

31. At a certain instant, an object flying horizontally to the right at 1 m s^{-1} suddenly explodes into two fragments of mass ratio 1: 2. If just after the explosion the more massive fragment flies to the right at 3 m s^{-1}, what would happen to the other fragment just after the explosion?

A. ***It would fly to the left at 3 m s⁻¹.***
B. ***It would fly to the left at 4 m s⁻¹.***
C. ***It would be momentarily at rest.***
D. ***It would fly to the right at 1 m s⁻¹.***

32. Explain why it is impossible to have the following explosion.

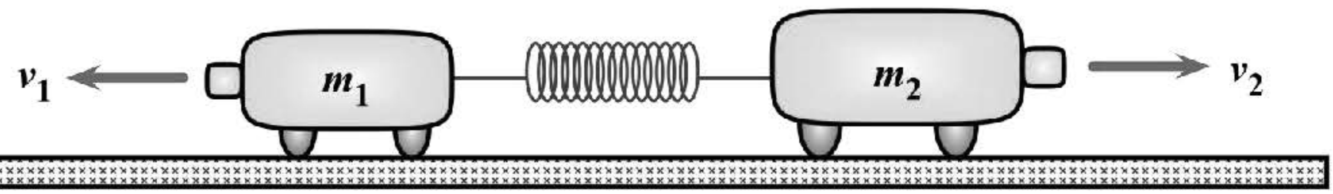
Since the total momentum of the system is ____________, this explosion is not possible.



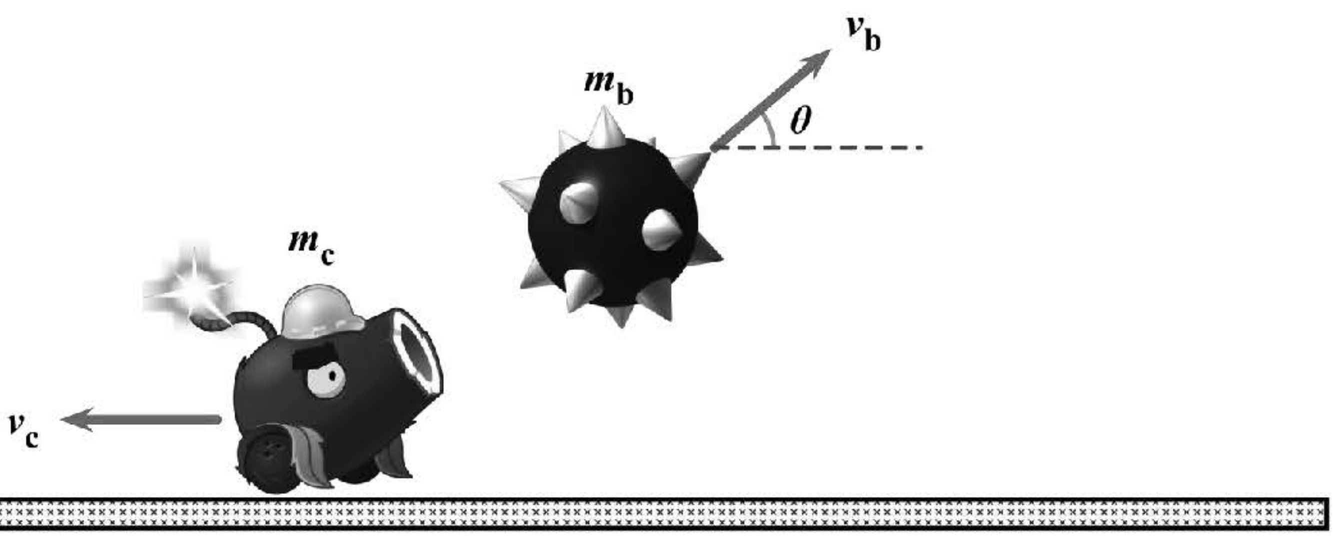
Typical examples of explosion:

Recoil of a rifle

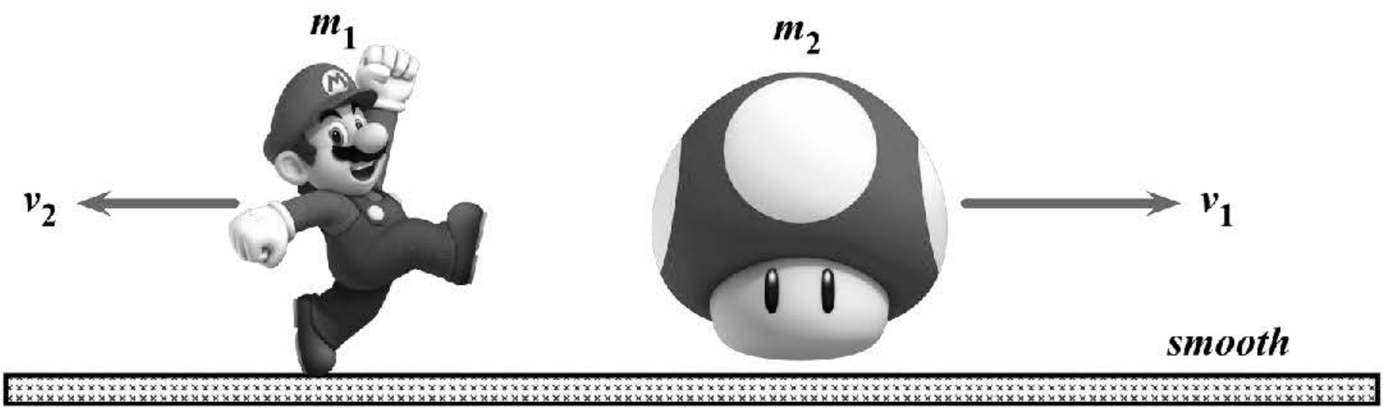
Release of a compressed spring



A cannon fires a cannonball

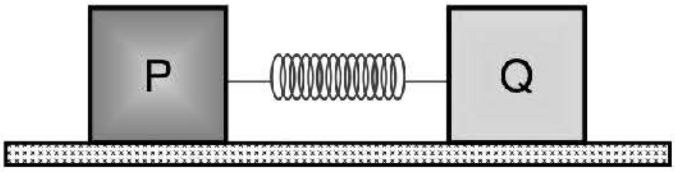


Mario throws away a mushroom



Examples that you must fully understand

33. A student compresses two blocks P of $0.8\ \mathrm{kg}$ and Q of $0.6\ \mathrm{kg}$ resting on a smooth surface against a spring and then releases them. After released, Q is found to move off with a velocity of $4\ \mathrm{m\ s^{-1}}$.



(a) What is the total momentum of the two trolleys after separation?

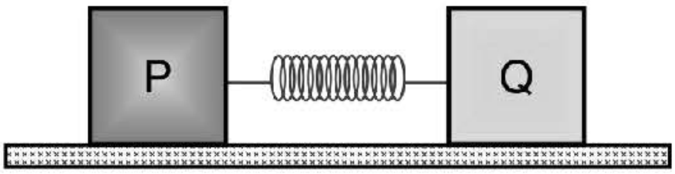
(b) Find the velocity of P after released.

(c) State the energy change.

The ____ energy of the student changes to ____ potential energy of the spring and then changes to ____ energy of the two ____.

(d) Calculate the elastic potential energy stored in the spring before released.

(e) Suppose P is held by the hand of a student during the release of the spring.



(i) What is the speed of Q after it moves off from the spring?

(ii) Is momentum of the two blocks conserved in this case? Explain briefly.

____! The momentum of the two blocks is not conserved since there is an ____ ____ acting on the two blocks by the hand.

Examples that you must fully understand

34. A rifle of mass $0.5\ \text{kg}$ fires a bullet of mass $10\ \text{g}$ with a muzzle velocity of $200\ \text{m s}^{-1}$ towards the right as shown below.

(a) Find the recoil velocity of the rifle when a bullet is fired.

(b) Which of the following statements is/are correct?

(1) The momentum of the bullet is equal to the recoil momentum of the rifle.

(2) The momentum of the bullet is equal in magnitude to the recoil momentum of the rifle.

(3) The velocity of the bullet is equal in magnitude to the recoil velocity of the rifle.

(c) Find the kinetic energy of the bullet and the rifle. Compare these two energies.

The kinetic energy of the bullet is ______ than that of the rifle since the bullet is ______ than the rifle.

(d) Suppose that in firing the rifle, $65\ \text{J}$ of energy is releases as heat, light and sound. Find the chemical energy stored in the bullet before explosion.

(e) Hence calculate the efficiency of the rifle in firing the bullet.

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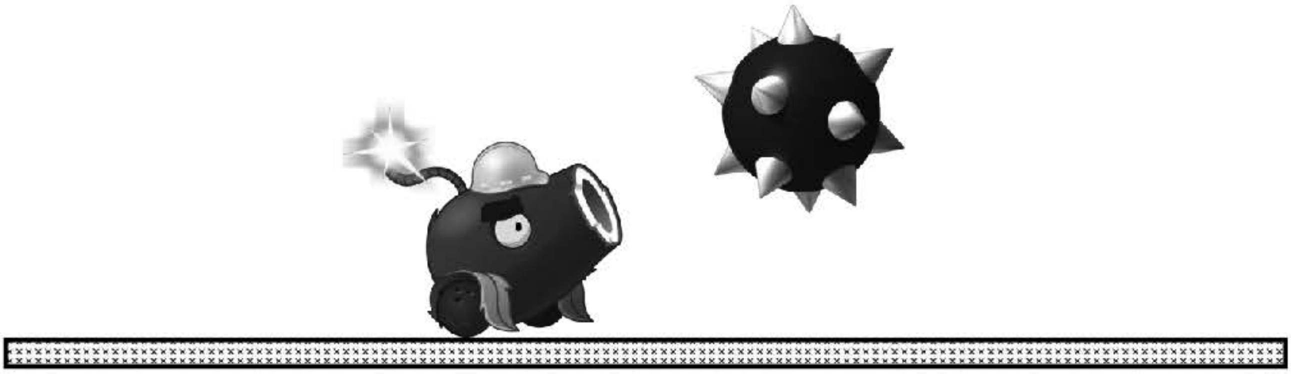
35. After firing a cannon ball of weight 500 N, the 6000 kg cannon then recoils at a speed of 3 m s^{-1}.



(a) Find the speed of the cannonball if it is fired at an angle of 30° above the horizontal.

(b) Jasmine said that the cannon applies a larger force on the cannonball that than the force acting on the cannon by the cannonball. Comment on this statement.

She is ______. Since the two forces form a ______________, according to Newton's Third Law, they have the same ______.

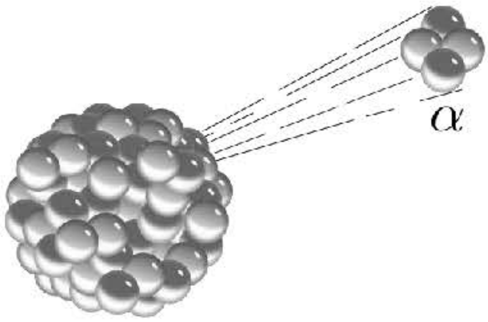
(c) Explain for the discrepancy between the chemical energy released by the explosive and the total kinetic energy of the cannon and the cannonball.

The ______ energy released is ______ than the total ______ energy of the cannon and the cannonball because some of the ______ energy released is converted into ______ and ______ energy.

Examples that you must fully understand

36. A Ra-226 nucleus emits an alpha particle when it undergoes an alpha decay according to the following equation:

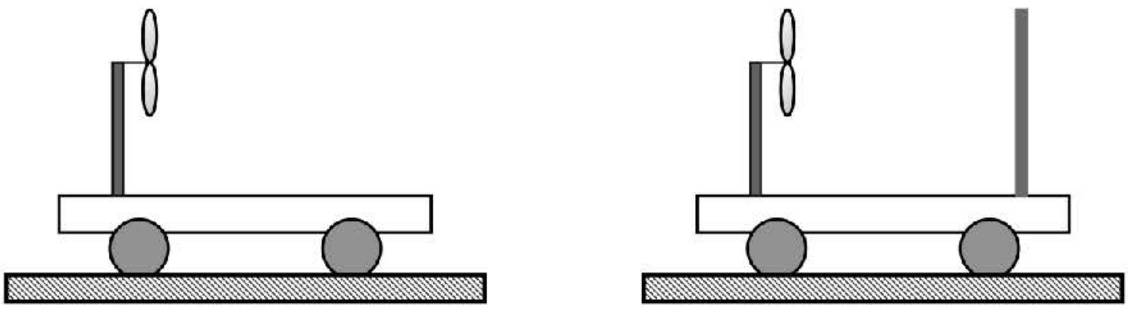
$$^{226}_{88}Ra \rightarrow ^{222}_{86}Rn + ^{4}_{2}\alpha$$



(a) Find the ratio of the velocity of the daughter nucleus to the velocity of the alpha particle.

(b) Find the ratio of the kinetic energy of the daughter nucleus to the kinetic energy of the alpha particle.

37. A fan is attached to a trolley initially at rest as shown in the above figure. When the fan is switched on and wind is blown towards the right, which of the following statements is / are correct?



(1) The trolley moves towards the left.

(2) The momentum of the trolley is conserved.

(3) The total momentum of the trolley and the wind is conserved.

(4) The force acting on the wind by the trolley is exactly equal in magnitude to the force acting on the trolley by the wind.

(5) If a sheet of cardboard is attached at the right-hand side of the fan, then the trolley will remain at rest.