Delta Science Education

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HKDSE Physics

Core 2: Force and Motion

Chapter 8: Gravitation

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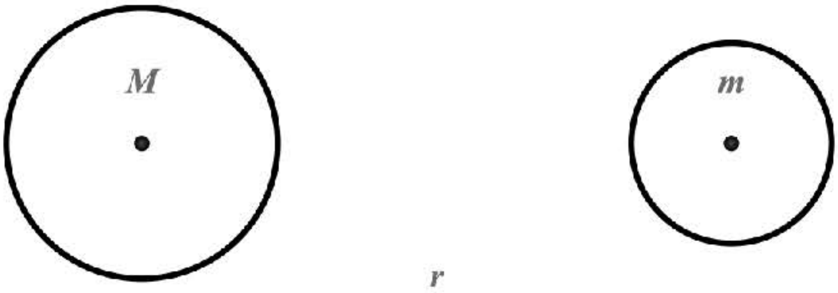
1. The Newton's Law of Universal Gravitation

(1) Statement of the Law of Gravitation

For two masses M and m, there exists a gravitational attraction force F which is proportional to the product of the two masses and inversely proportional to the square of the distance r between the two masses.



Universal Gravitational constant : $G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$

- The distance should always be measured from the centre of mass of the bodies.
- The two forces form an action and reaction pair according to Newton's Third Law.
- Note that gravitational force must always be attractive.

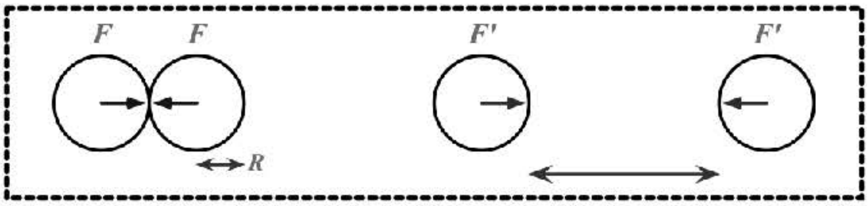
Examples that you must fully understand

1. Calculate the gravitational force acting on Donald Tsang whose mass is 198.56 kg.
   (Given $M_E = 6.37 \times 10^{24}$ kg, $R_E = 6400$ km)

Examples that you must fully understand

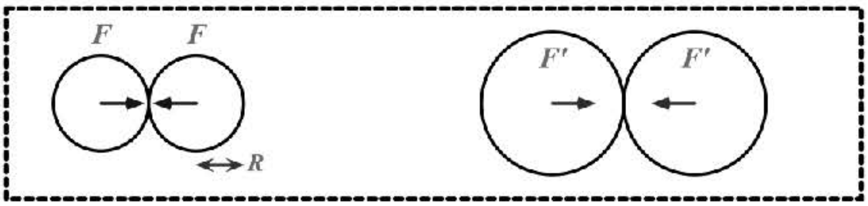
2. The gravitational force between two identical spheres in contact, each of radius R, is F.

(a) What is the gravitational force if one of the spheres is displaced by a distance of $6R$ away?



(b) What is the gravitational force if the two spheres are replaced by two made of different material having a density three times of the original material (the size of the spheres remains unchanged)?

(c) What is the gravitational force if the two spheres are replaced by another two made of the same material but with radius $3R$ and are still in contact?

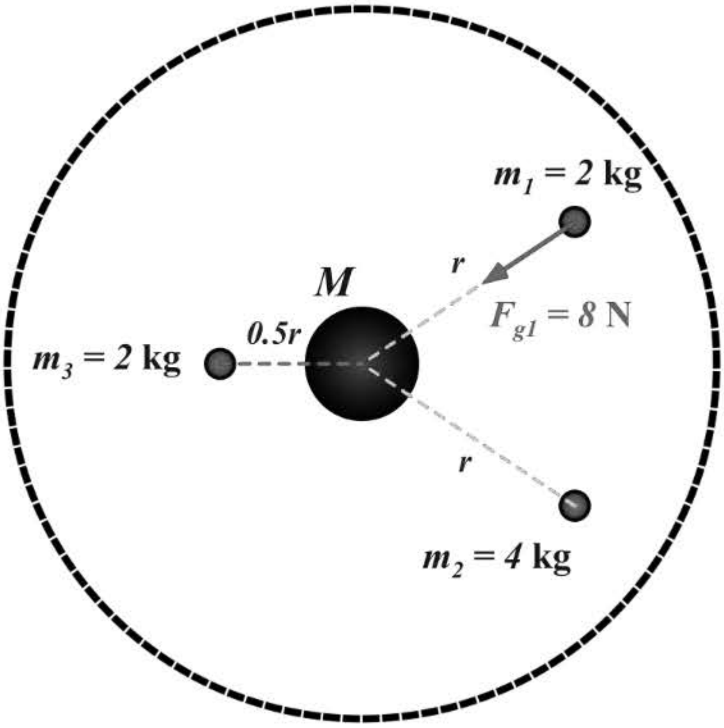


(d) What is the gravitational force if both of the spheres are replaced by another one made of material with triple density and the five times radius?

2. Gravitational field

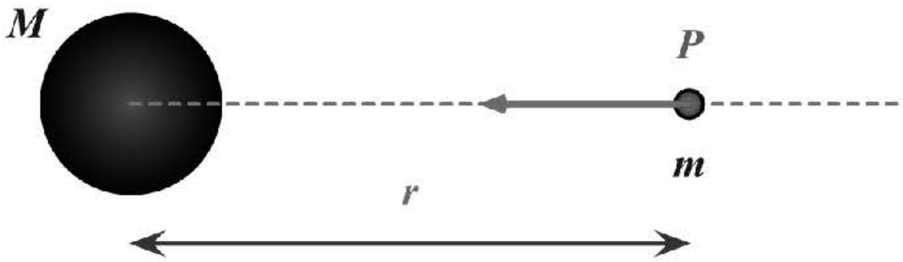
(1) Definition of gravitational field strength

The gravitational field at a point is the gravitational force acting on a unit mass placed at that point.

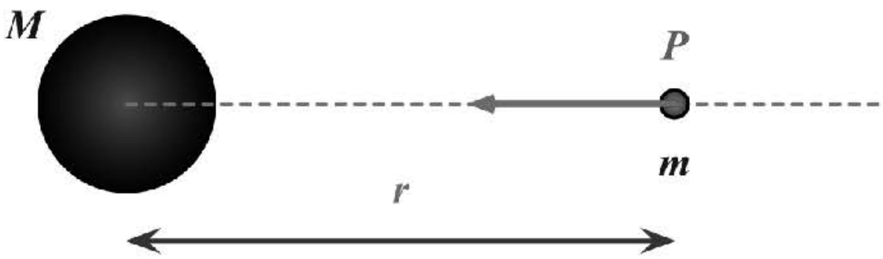


- Consider an object free falling near the surface of the Earth:

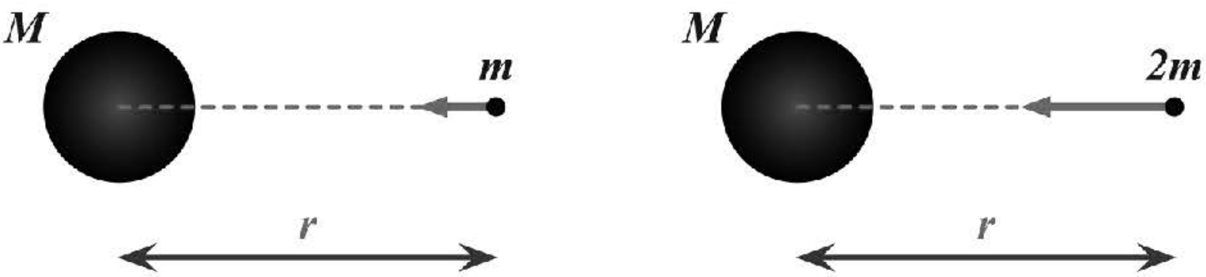
(2) Gravitational Field strength around a body of mass M



(3) Properties of Gravitational Field

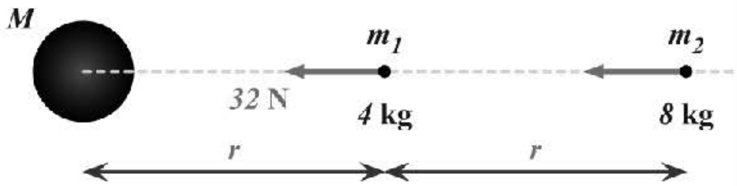


- Gravitational field is a vector.
- Gravitational field must always direct towards the centre of the body M.
- Field strength depends only on the mass of central object and the position r.



Examples that you must fully understand

1. Two small masses m_1 of 4 kg and m_2 of 8 kg are placed at distance r and $2r$ from a large mass M as shown below. The gravitational force experienced by m_1 is 32 N.

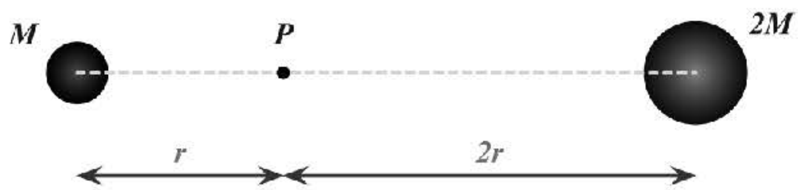


   (a) What is the gravitational field at a distance of r?

   (b) What is the gravitational field at the distance of $2r$?

   (c) What is the gravitational force acting on the mass m_2?

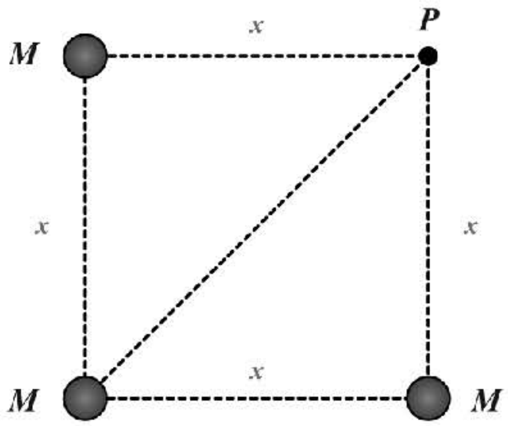
2. Two masses M and $2M$ are placed as shown below.



   (a) Find the gravitational force between the two masses.

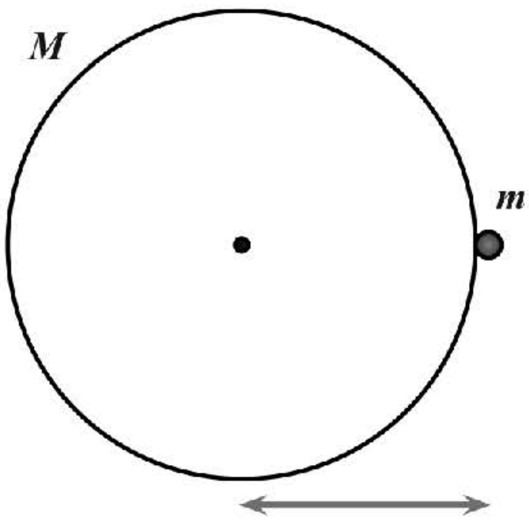
   (b) Find the gravitational field at the point P.

3. Find the gravitational field strength g at point P as shown in the diagram below.



3. Gravitational field of the Earth

(1) Field strength at the Earth's surface



- Assume the Earth is a uniform sphere, thus, the field strength at the Earth's surface is

- Given:

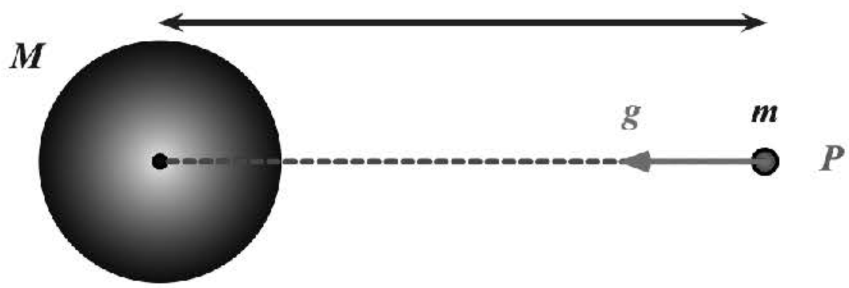
$M_E = 6.0 \times 10^{24}$ kg

$R_E = 6.4 \times 10^{6}$ m

$G = 6.67 \times 10^{11}$ N m^2 kg^{2}

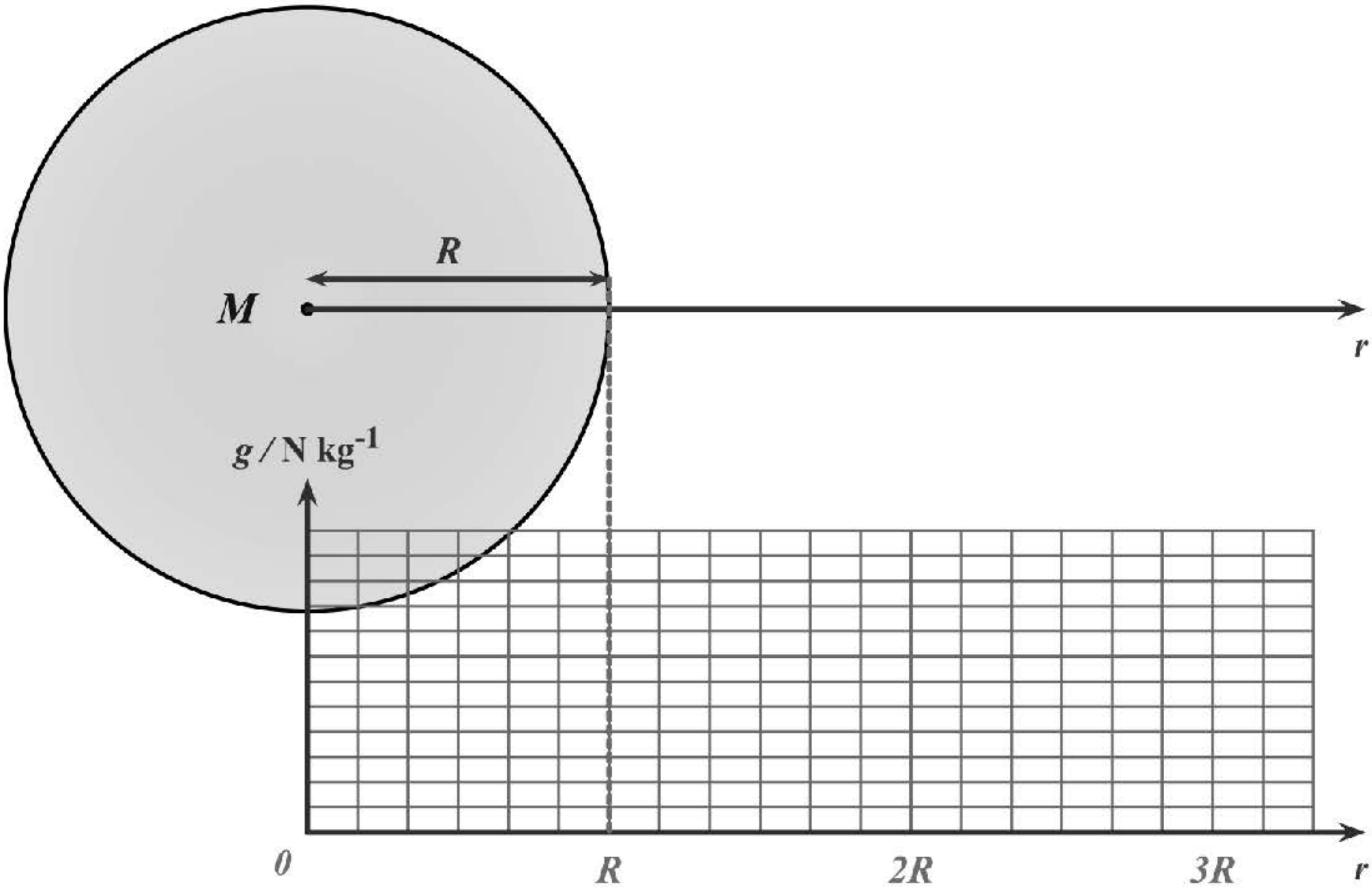
$\therefore g =$

(2) Field strength above the Earth's surface



- The gravitational field strength at ***P***:

(3) Variation of the Gravitational Field of the Earth

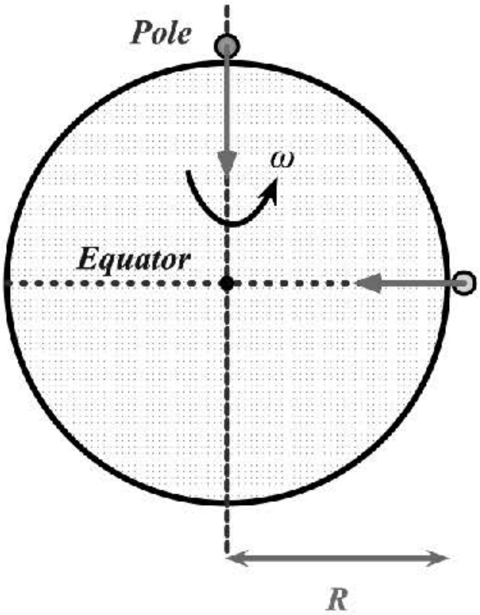


Fxamples that you must fully understand

Given that the Earth's radius is 6 400 km. A satellite is at a radius of 3 200 km from the Earth's surface. Find the gravitational field strength at that orbit.

Method 1	
Method 2	

(4) Effect of self-rotation of the Earth on the field



■ Assume the Earth to be a perfect sphere, having a self-rotation period as **24 hours**.

At the equator	At the poles
✓ Part of the gravitational force is used to provide the centripetal force for circular motion. ✓ The apparent weight mg_E is smaller, thus the g_E is smaller.	✓ Since the poles stay at rest, thus the field strength at the poles is

Examples that you must fully understand

1. Assume the Earth is a perfect sphere with a radius of 6 400 km. A boy of mass 100 kg is found to have a balance reading of 980 N at the north pole.

   (a) Calculate the difference of gravitational field between the pole and the equator.

   (b) Hence find the balance reading measuring the weight of the boy at the equator.

Examples that you must fully understand

2. If you were the God, calculate how fast the Earth should be rotating so that people on the equator experience ZERO weight. You should give your answer in period (in hour).

3. There is a planet very similar to the Earth. However, its radius is double that of the Earth and the self-rotational speed is also double that of the Earth. Molly of mass $70\ \mathrm{kg}$ is found to have a balance reading of $1\,373.4\ \mathrm{N}$ at the North pole of the planet. Calculate the balance reading of her at the equator.

4. A body is placed at the equator of the Earth. Its weight is measured by a balance. Which of the following statements concerning the balance reading is/are correct?

   (1) ***The balance reading is increased gradually when the body is brought from the equator to the North pole.***

   (2) ***When the body is at the equator, the balance reading would decrease gradually if the self-rotational speed of the Earth is increased gradually.***

5. Taking the Earth to be a perfect sphere of uniform density rotating about its polar axis, which of the following statements concerning the observed acceleration due to gravity, g, at the surface of the Earth is true?

   (1) ***g at the equator is smaller than that at the poles.***

   (2) ***If the rate of rotation of the Earth slows down, g at the equator increases.***

   (3) ***If the radius of the Earth increases with its density remaining unchanged, g at the poles decreases.***

(5) Relation of the field strength with the density

Density	Volume	Gravitational Force	Field Strength
✓ Assume the Earth has a uniform density	✓ Assume the Earth to be a perfect sphere		

Examples that you must fully understand

1. The moon has a density about **63%** of the Earth. It is known that the gravitational field strength at the Moon's surface is about one-sixth of that of the Earth. Given that the radius of the Earth is **6 400 km**, estimate the radius of the Moon.

2. An object of mass **8 kg** has a weight of **25 N** on the surface of planet *X*. Radius of the planet *X* is **2 400 km**. Given that the universal gravitational constant is 6.7×10^{-11} N m^2 kg^{-2}. What is the mass of the planet *X*?

3. A planet has a diameter **3.6 times** that of the Earth but the Earth has a density **2 times** that of the Planet. What is the gravitational field strength on the planet's surface?

Examples that you must fully understand

4. The gravitational field strength at the surface of a certain planet is $9.5\ \text{N kg}^{-1}$. Given that the universal gravitational constant is $6.7 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$. The average density of the planet is $6500\ \text{kg m}^{-3}$.

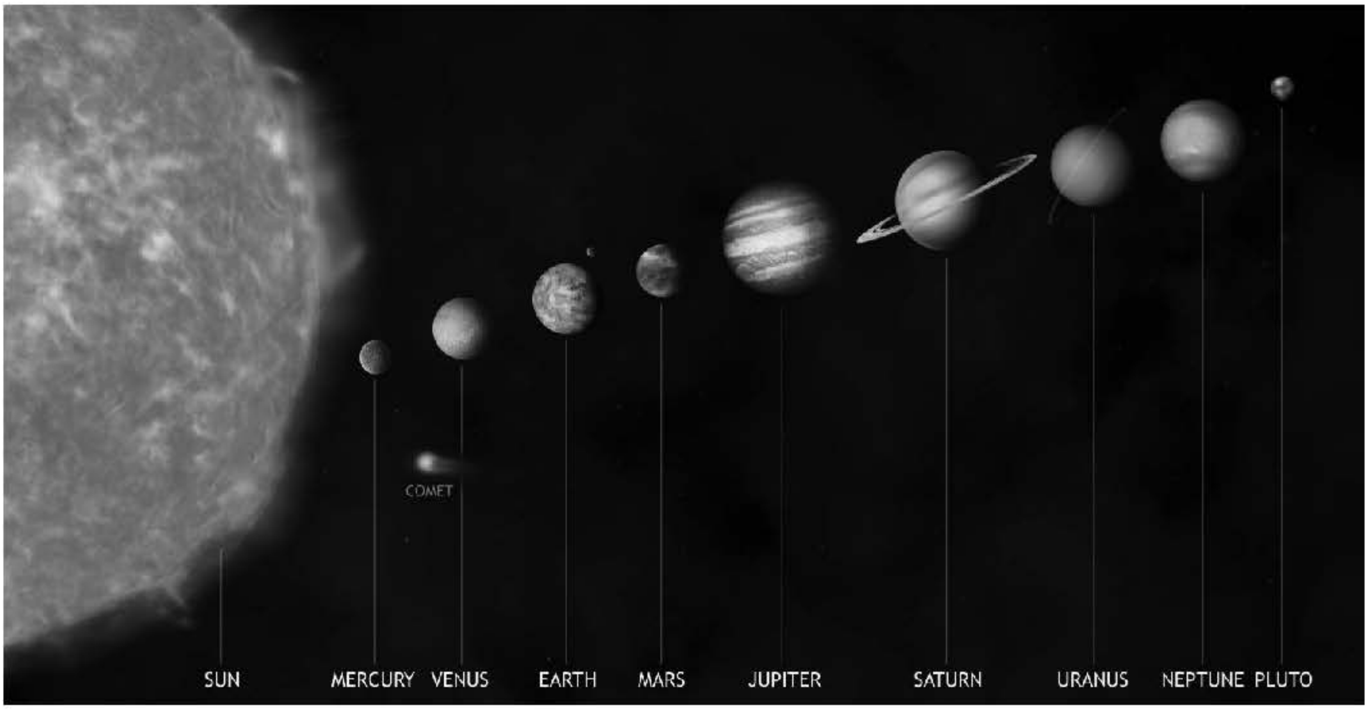
(a) Calculate the radius R of the planet.

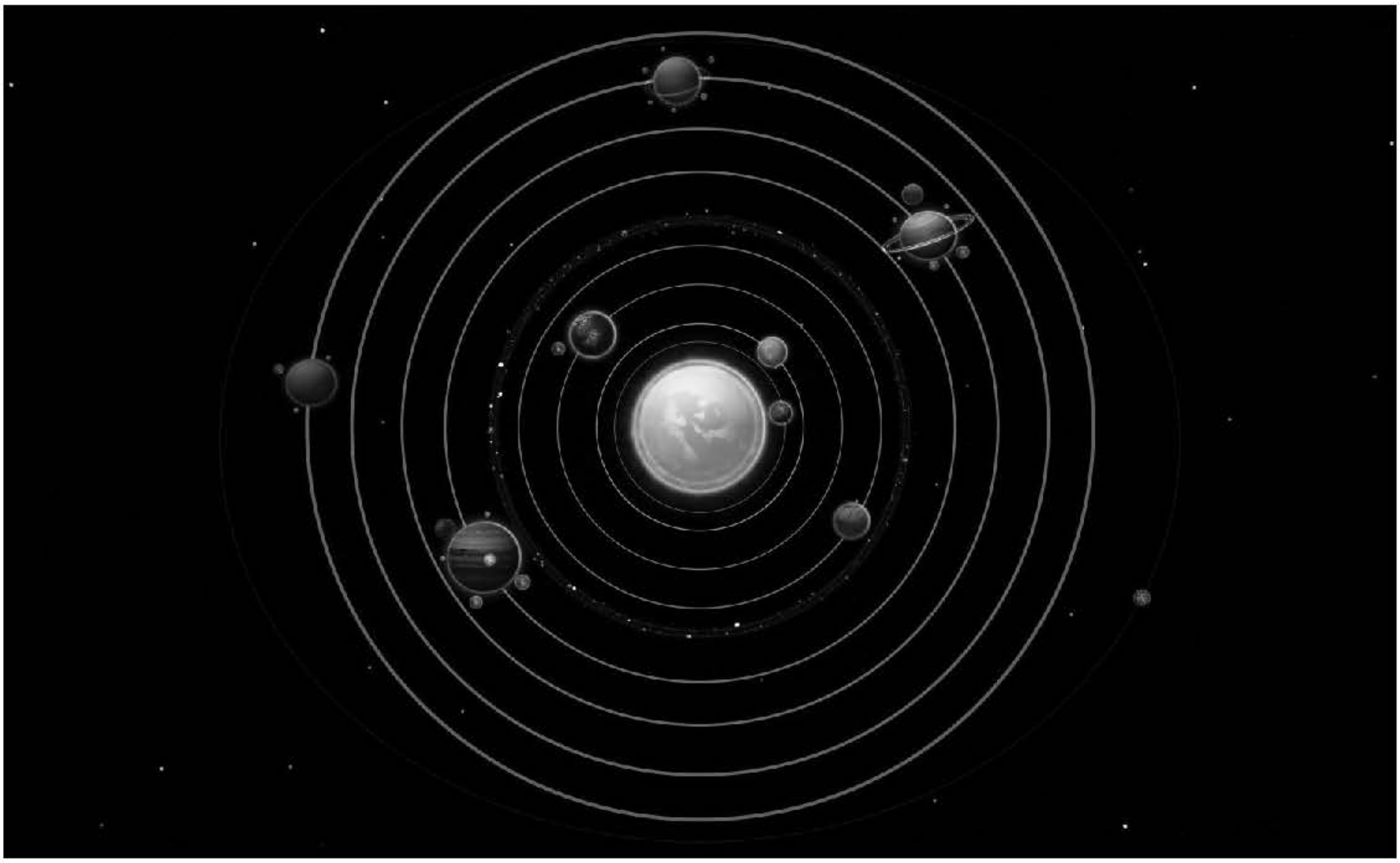
(b) What is the gravitational field strength at a height of $950\ \text{km}$ above the planet's surface?

5. Given the radius of the Earth to be $6\,400\ \text{km}$ and the fact that $g_0 = 10\ \text{N kg}^{-1}$ at the Earth's surface, find the average density of the Earth.

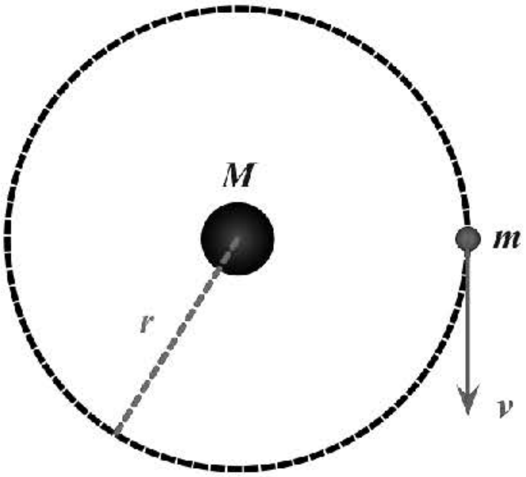
4. Orbital motion of Astronomical bodies

(1) Use Newton's Law of Gravitation in astronomy





- In a simple solar system, a star is an astronomical body assumed to be stationary.
- Planets move round a star in circular orbit while satellites move round a planet in circular orbit.
- When an object m moves in circular orbit round a central astronomical body M, the gravitational force provides the centripetal force for the circular motion of the object:



For an object in orbital motion:

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1. Calculate the gravitational field and the gravitational force experienced by Kepler-62e which is a planet very similar to the Earth.

Given:

Mass of Kepler-62e $= 36M_E$ $1M_E = 5.97\times10^{24}\,kg$

Average radius of its orbit $= 0.42AU$ $1AU = 1.5\times10^{11}\,m$

Orbital period $= 122.3874$ Earth Day

2. Mercury is the planet closest to the Sun in the Solar system. It is known that the time taken for Mercury to revolve the Sun once is 88 days. Given that the mass of the Sun is 2×10^{30} kg and $G = 6.7 \times 10^{-11}$ N m^2 kg^{-2}.

(a) What is the average distance of Mercury form the Sun?

(b) State TWO assumptions in the above calculation.

The Sun is ☐ and the orbit is a perfect ☐.

(c) What is the linear speed of Mercury?

(d) Is the gravitational force acting on the Sun by the Mercury is greater than, equal to, or smaller than that acting on the Mercury by the Sun? Please explain your answer.

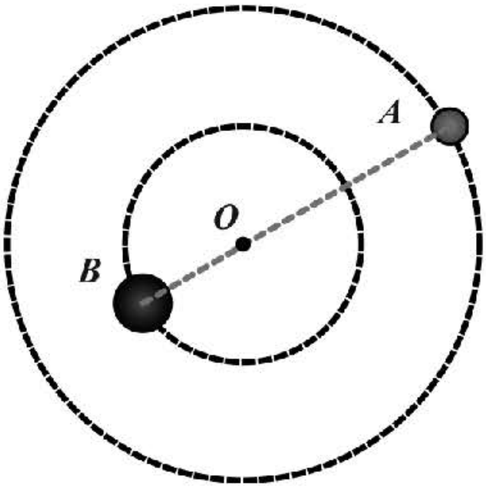
The gravitational force acting on the Sun by the Mercury is ☐ ☐ that acting on the Mercury by the Sun, since they are ☐ ☐ ☐ ☐.

(e) The effect of the gravitational force acting on the Mercury enables it to move in circular motion. What is the effect of the similar gravitational force on the Sun?

The effect on the Sun is ☐ since the mass of the Sun is ☐ ☐ than that of the Mercury.

Examples that you must fully understand

3. In a binary star system, two stars A and B are revolving about their centre O with uniform circular motion under their mutual gravitational force. If the radius of the orbit of A is twice that of B, which of the following statements is/are correct?



(1) The force acting on A is smaller than that on B.

(2) The angular speed of A is equal to that of B.

(3) The orbital speed of A is twice that of B.

(4) The centripetal acceleration of A is twice that of B.

(5) The mass of A is twice that of B.

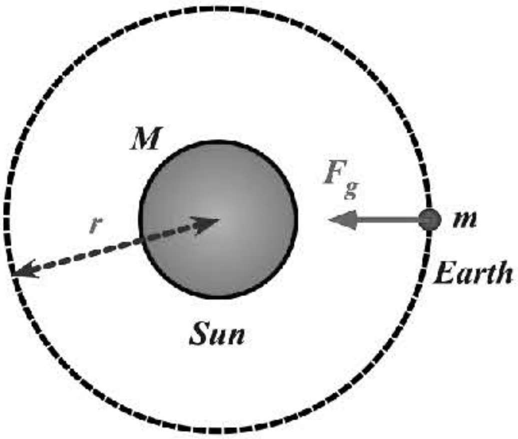
4. Given $G = 6.7 \times 10^{-11}$ N m^2 kg^{-2}.

(a) **The Earth moves round the Sun with a radius of 1.5×10^{11} m measured from the centres. Calculate the mass of the Sun.**

(b) **A satellite moves round the Earth with a height of 9.5×10^6 m from the Earth's surface and a period of 5.52 hours. Calculate the mass of the Earth. Given the radius of the Earth is 6.4×10^6 m.**

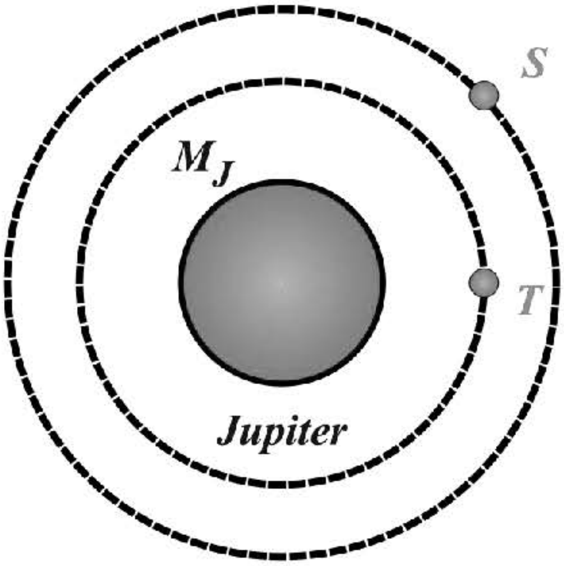
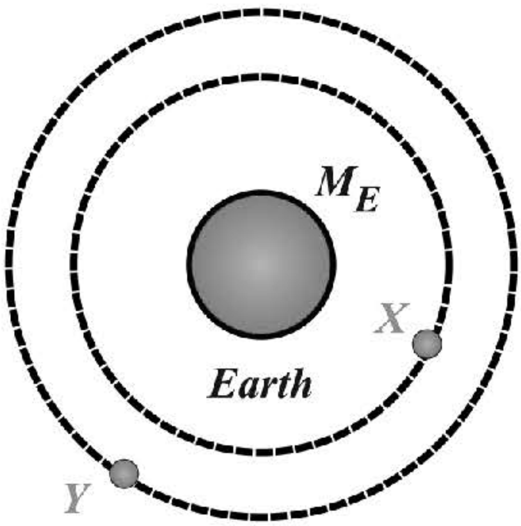
(2) Kepler's Third Law

- Consider the Earth is revolving round the Sun in a perfect circular orbit with the Sun right at the centre:



- Kepler's third law states that the square of the period of revolution of a planet is proportional to the cube of its mean distance from the sun.

- Please note the constant depends on the mass of the object at the centre.

Examples that you must fully understand

1. When a body is thrown upwards with a speed u at a planet's surface, it takes time t to fall back to the original position. Now if the body is given a speed v so that it revolves around the planet's surface in circular motion, what is the radius R of the Earth?

2. A spacecraft of mass 7.8×10^3 kg moves in a circular orbit around the Earth. It revolves round the Earth 77 times, travelling a total distance of 3.25×10^9 m. (Radius of the Earth is 6.37×10^6 m.)

   (a) Calculate the radius of the orbit of the spacecraft.

   (b) Calculate the speed of the spacecraft in the orbit.

   (c) How long does it stay in the orbit?

3. Two satellites of the same mass travel around the Earth in circular orbits of different radii. The satellite in the orbit with greater radius has

   (1) a greater speed

   (2) a greater angular speed

   (3) a longer period

   (4) a greater acceleration towards the Earth's centre

Examples that you must fully understand

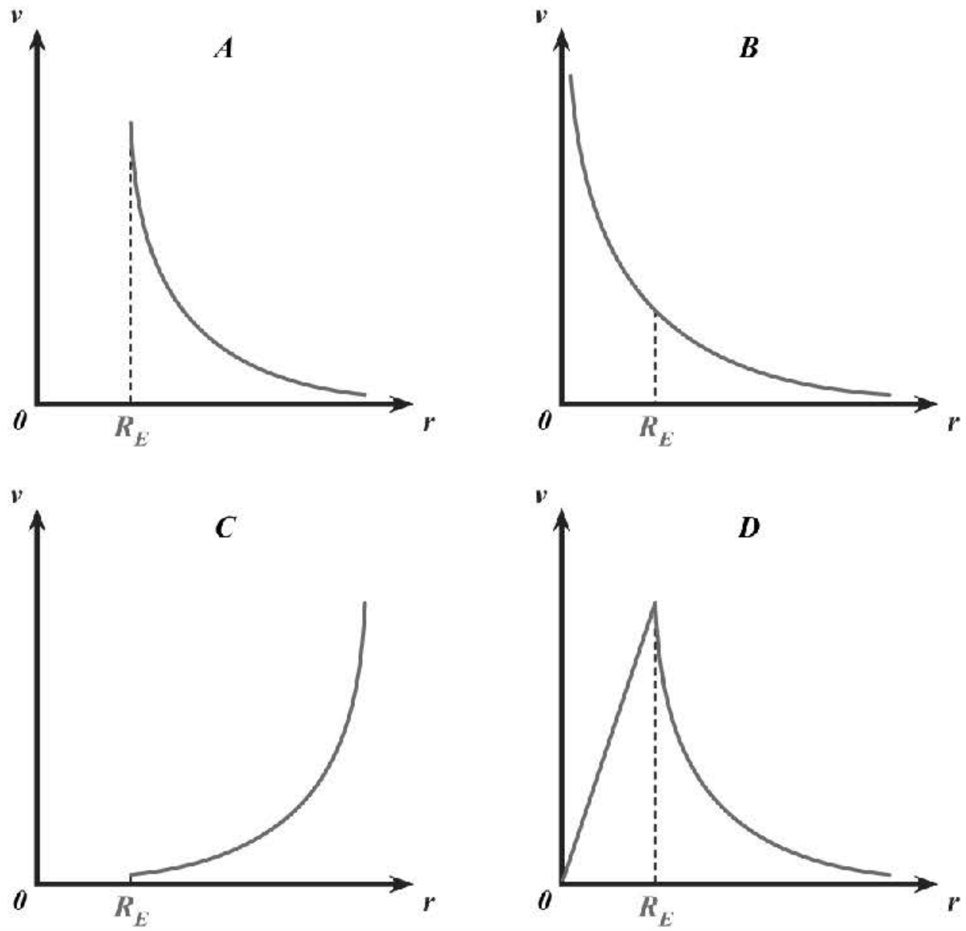
4. A satellite A orbits round the Earth once every **40 days** with a mean radius of R. What is the period of another satellite B having a mean radius of $4R$?

5. Hence, if there is another satellite C which wants to have a period of **10 days**, what orbital radius should it have?

6. An old communication satellite is replaced by a new satellite which has twice the mass of the first one. Each of the satellite has an orbit period of **24 hours**. What is ratio of

$$\frac{\text{radius of the orbit of the new satellite}}{\text{radius of the orbit of the old satellite}} = ?$$

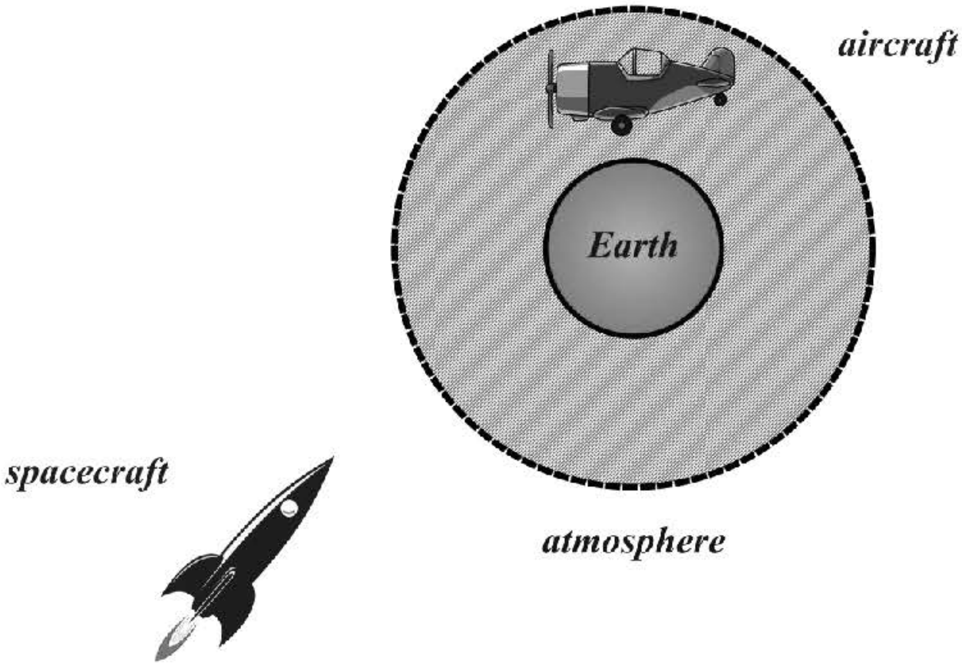
7. If v is the speed of a satellite which revolves round the Earth in a circular orbit of radius r, which one of the following graphs best represents the variation of v with r? (R_E is the radius of the Earth)



Examples that you must fully understand

8. One of Jupiter's moons is called Garymede. The radius of its orbit around Jupiter is about **3 times** that of the Moon around the Earth. The period of the Moon around the Earth is about **3.4 times** that of the Ganymede around Jupiter. If the mass of the Earth is M_E, what is the mass of the mass of the Jupiter?

9. A spacecraft has just finished its mission and it is planning to return to the Earth. What should be the physical properties of the surface material that the spacecraft made of and why?



(1) It should have ____ melting point to prevent the spacecraft from ____.

(2) It should have large ____ ____ ____ so that temperature rise is small.

(3) It should have ____ heat ____ to prevent heat ____ into the spacecraft.

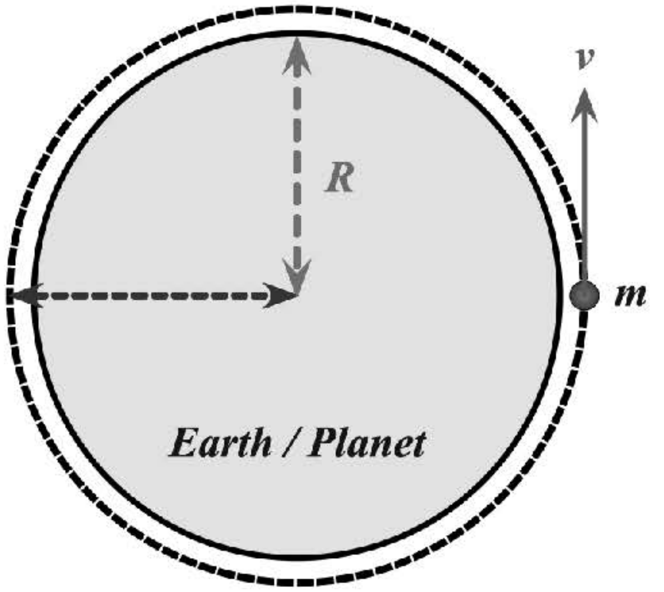
Hence, give TWO reasons why an aircraft is unable to fly in space like a spacecraft fired with rocket.

(1) Aircraft needs ____ to provide a ____ force.

(2) Aircraft draws ____ into the engine for ____ of the fuel.

(3) Low-altitude satellite

- A low-altitude satellite (close orbit satellite or spy satellite) moves round the Earth in an orbit close to the Earth's surface.



Points to note:

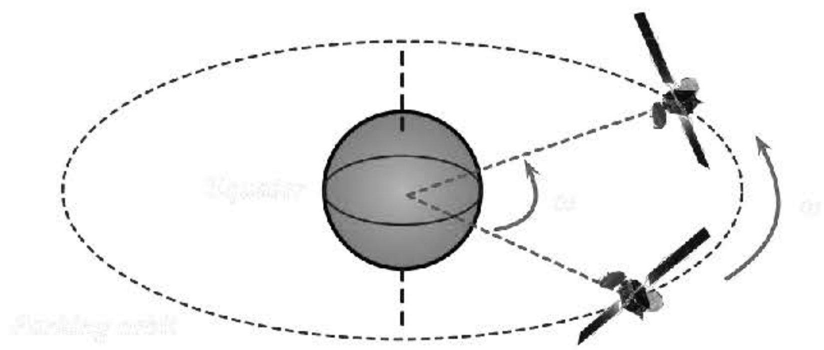
1.

2.

- Due to the atmosphere, the energy of this satellite would gradually be dissipated into internal energy.

(4) Parking satellite

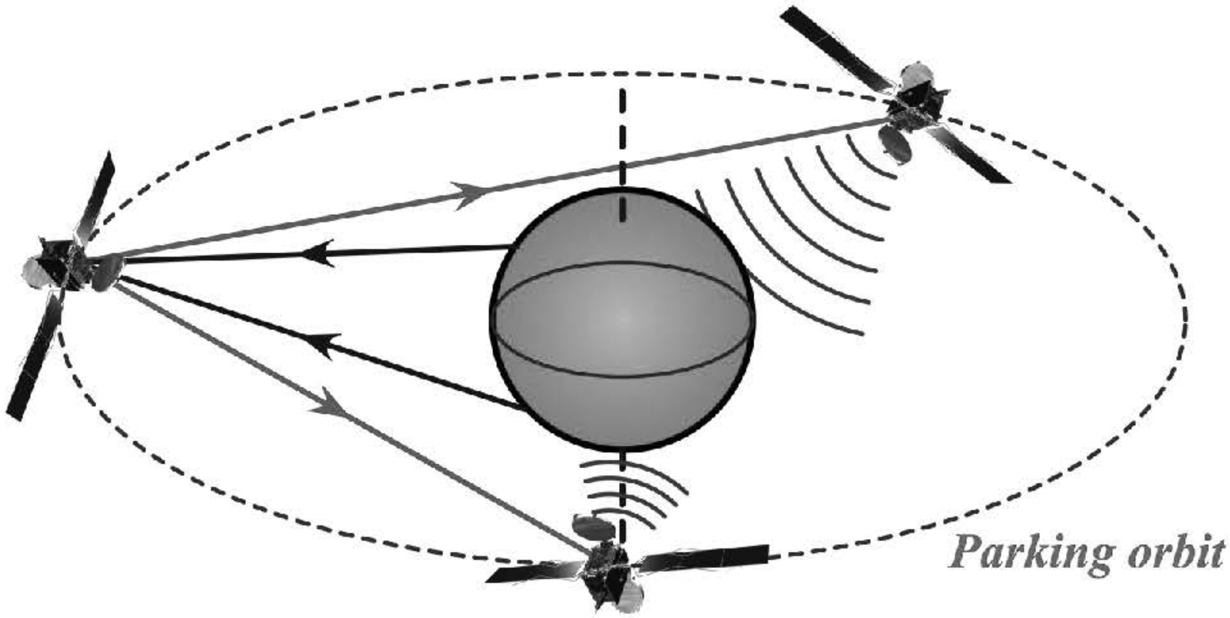
- A parking satellite (geostationary satellite, synchronous satellite or communication satellite) seems to be stationary above a certain position above the Earth.

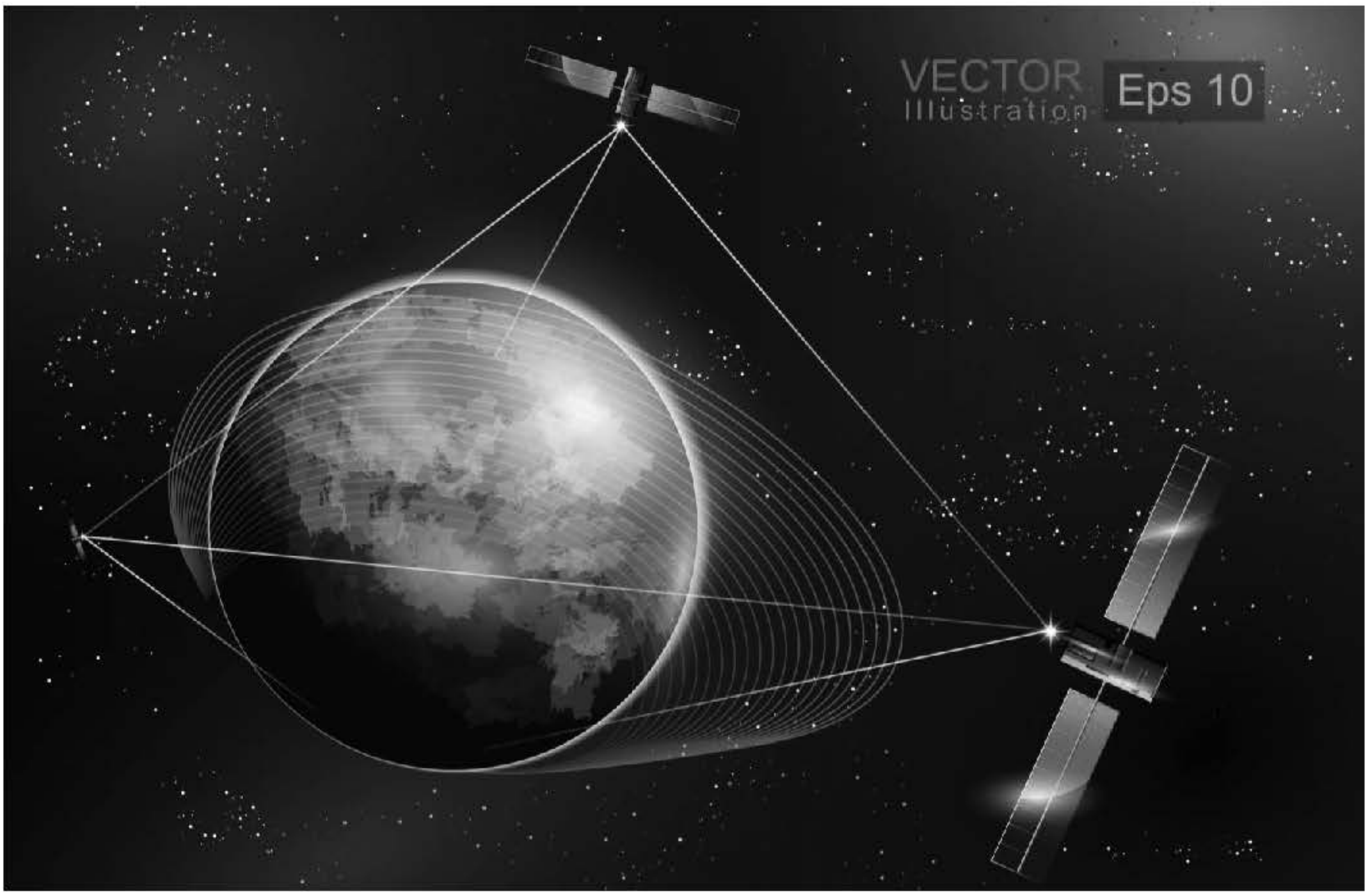


Points to note:

1. Must be at the equatorial plane.
2. Must rotate in the same sense of the Earth with the same period/angular speed.
3.
4.

- Parking satellites are used in satellite communication. It transmits microwave signals from one place to another.
- The satellites receive the microwave signal (microwave in the order of 10^{-2} m) from one ground station and then re-transmits the signal to another point at the Earth's surface.
- About one third of the Earth's surface can be covered.

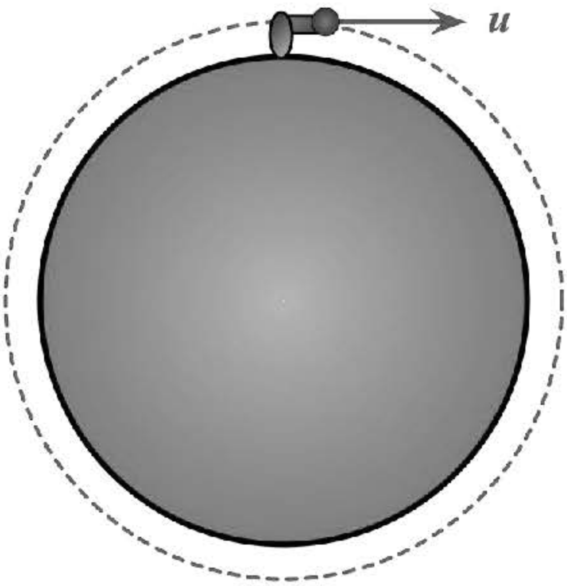




Examples that you must fully understand

1. A low-altitude satellite near the Moon's surface has a speed of $1.77\ \text{km s}^{-1}$. The radius of the Earth is about **4 times** that of the Moon and the ratio of the average density of the Earth to that of the Moon is about **5 : 4**. What is the speed of a low-altitude near the Earth's surface?

2. Assume the Earth to be a non-rotating sphere, Newton once argued that if a cannon-ball were fired horizontally at a very high speed from any point on the Earth, it would eventually return and strike the cannon from behind.



(a) If the speed of the cannon-ball remains constant, what would be the least time for it to arrive at the cannon again? (You can take the radius of the Earth to be 6.37×10^6 m)

(b) Give reasons that the cannon-ball in fact cannot arrive at the cannon.

(1) The Earth is not a ______ ______, thus g would vary over the Earth's surface.

(2) There is ___ ______ acting on the ball, thus its speed cannot be ______.

(3) The Earth ______, thus the cannon has moved to a different ______.

(4) The ball may hit ___ ______.

Examples that you must fully understand

3. *X* and *Y* are two planets. Each of them has a low-altitude satellite revolving in a circular orbit close to the planet. If the two satellites are observed to have the same period, then *X* and *Y* must have nearly the same

(1) ***mass***

(2) ***average density***

(3) ***radius***

(4) ***acceleration due to gravity at the planet's surface.***

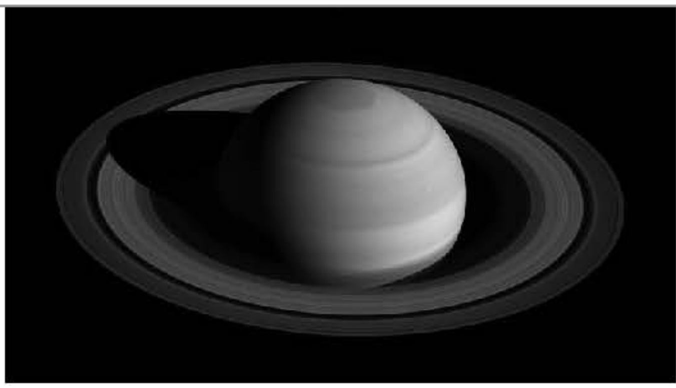
4. *X* and *Y* are two planets. Each of them has a low-altitude satellite revolving in a circular orbit close to the planet. If the two satellites are observed to have the same density, which of the following statements concerning the two satellites is/are correct?

(1) ***The two satellites must have the same speed.***

(2) ***The two satellites must have the same acceleration.***

(3) ***The two satellites must have the same period.***

5. Long ago, scientists wanted to find out whether the right of Saturn is a rigid body of a group of satellites revolving around the Saturn. The linear speed v of different layers of the ring were measured and how v varies with the distance r from the centre of the Saturn was determined. Which of the following relations between v and r supports the supports the suggestion that

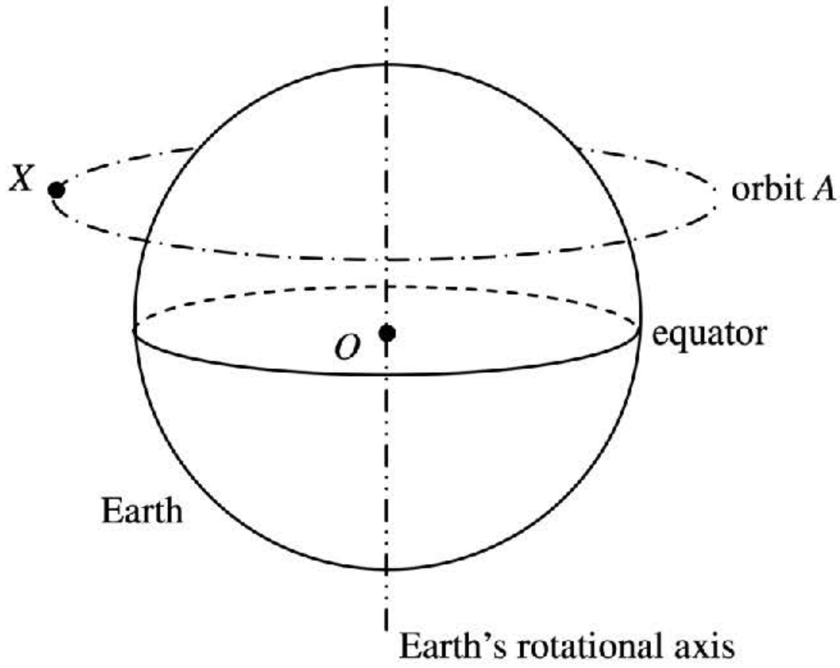


(1) the ring is a rigid body,

(2) the ring is a group of satellites revolving around the Saturn.

Examples that you must fully understand

6. In the figure below, X is a point in space and O is the centre of the Earth.



(a) Explain the meaning of the parking orbits for satellites. How can it be achieved?

These satellites appear ______ relative to observers at the Earth's surface. It can be achieved by ______ round the Earth with the same ______ of the Earth and in the same rotation ______ of the Earth.

(b) Briefly explain why the satellite cannot move in a circular orbit A as shown under the influence of the Earth's gravitational force only.

The direction of the ______ centripetal force is different from the direction of the ______ force acting on the satellite. The ______ ______ of the gravitational force will ______ the satellite towards the equator.

7. Given that the radius of the Earth is 6 370 km. Calculate the height of a communication satellite in the parking orbit.

Examples that you must fully understand

8. Which of the following statements is/are correct?

(1) A satellite must be far away from the Earth so that it is not affected by the Earth's gravitational field.

(2) There is a communication satellite directly above Hong Kong.

(3) There is more than one parking orbit above the Earth's surface for communication satellite.

(4) The acceleration acting on a parking satellite always points towards the centre of the Earth.

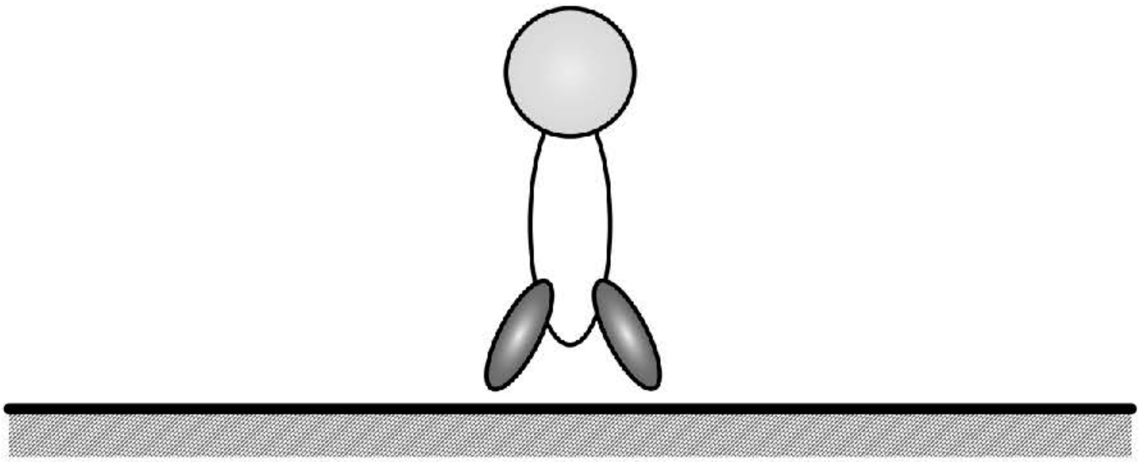
(5) A communication satellite is always vertically above the same place on the Earth's surface.

(6) A geostationary satellite must be rotating in the same sense and with the same angular speed as the Earth.

5. Weightlessness

(1) Feeling of weight

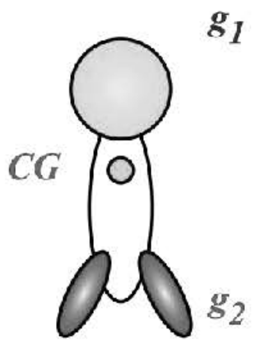
- Feeling of weight is due to the normal reaction acting on our body from the ground.



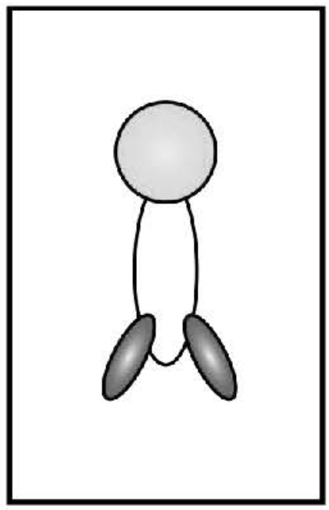
- Whenever there is no normal reaction acting on a man, he feels weightlessness.

There is no gravitational force.	Gravitational force is used to provide acceleration.
• At a position where the g due to one astronomical body is balanced by the g due to another astronomical body. • At a position far from any astronomical body. ➢ **No weight and normal reaction act on the object.**	• For an object under free fall, its weight is used to provide the acceleration. • For an object orbiting round the Earth, its weight is completely used to provide the centripetal force (acceleration) required for circular motion. ➢ **No normal reaction acts on the object.**

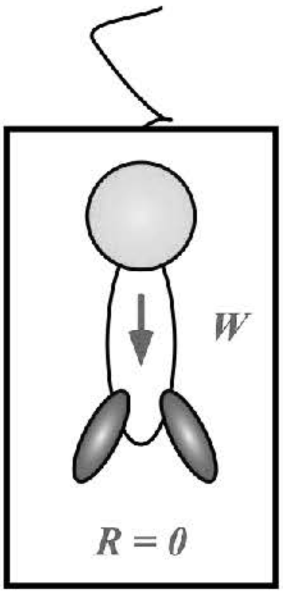
An object at a point between two astronomical bodies



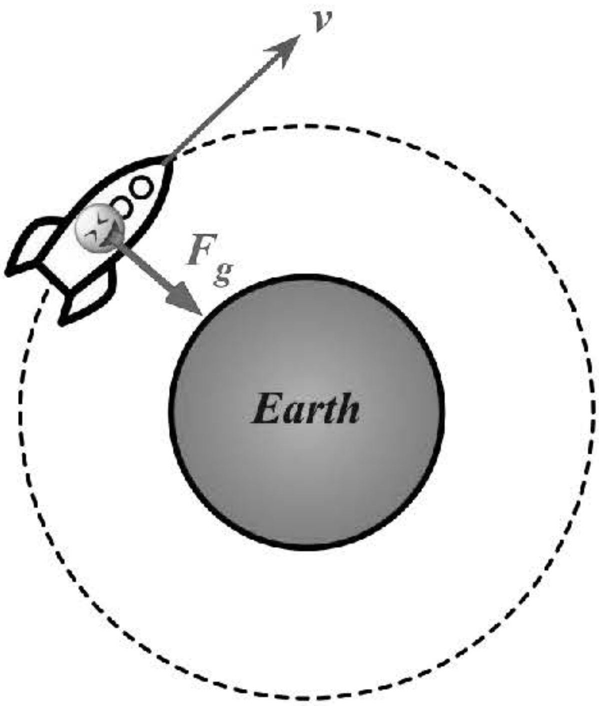
An object in far space



An object under free fall



A spacecraft moving round the Earth



Examples that you must fully understand

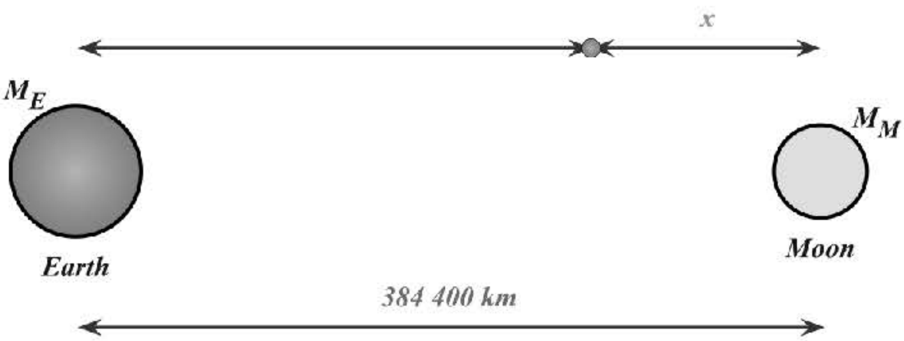
1. An astronaut is inside a satellite orbiting round the Earth. He experiences weightlessness during the orbiting motion. This can be explained by:

   (1) the gravitational pull of the Earth is exactly cancelled by the gravitational pull of the Moon.
   (2) the gravitational attraction between the astronaut and the Earth is just sufficient to provide the centripetal force which keeps him in orbit.
   (3) the is no reaction force acting on the astronaut by the ground of the spacecraft.

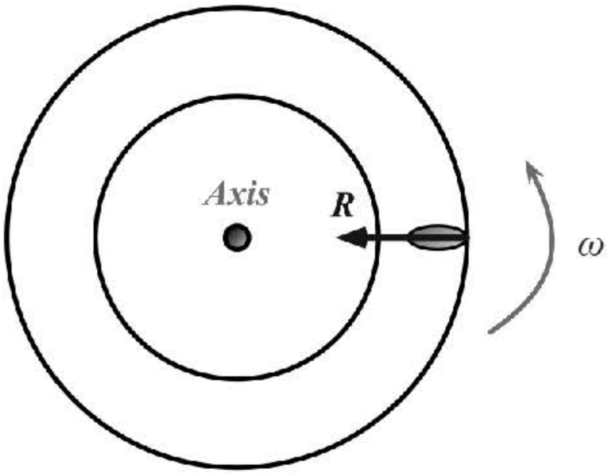
2. In which of the following situation is the magnitude of the normal reaction of the supporting surface always equal to the weight of the body?

   (1) A ball bouncing vertically on a horizontal ground is in contact with the ground.
   (2) An astronaut in a spacecraft which performs circular motion around the Earth.
   (3) A boy standing in a lift which is moving vertically upward with a uniform velocity.

3. There is a point along the line of the Earth and the Moon where the gravity is zero. Calculate the distance of this point from the Moon. Given that the mean Earth-Moon distance = 384 400 km, $M_E = 5.97 \times 10^{24}$ kg and $M_M = 7.34 \times 10^{22}$ kg)

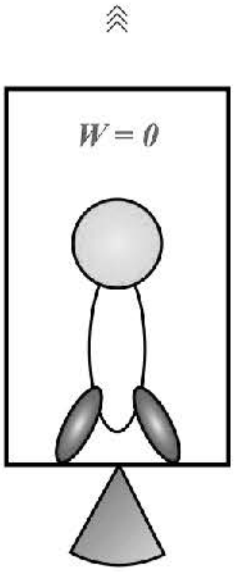


(2) Artificial gravity



Top view

- Artificial gravity can be created by rotating the space station about a fixed axis with a constant angular speed such that the magnitude of the centripetal acceleration is equal to the magnitude of desired gravity.
- Normal reaction force acts on the astronauts to provide the centripetal force for circular motion. Thus, he experiences the feeling of weight.



Examples that you must fully understand

A donut-shaped space station is far away from any astronomical objects. It is designed such that people living at the periphery experience an artificial gravity similar to that on the Earth. If the radius of the periphery from the centre is 700 m, what should be the angular speed of the space station?